Brand Reallocation and Market Concentration

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Abstract

We study the interaction of customer capital and productivity through brand reallocation across firms. We develop a firm dynamics model with brands as transferable customer capital, heterogeneous firm productivity, and variable markups. We study the matching process between transferable brand capital and core productivity, which can be inefficient with significant welfare implications. We link USPTO trademark data with Nielsen sales data to study the prevalence of brand reallocation and the response of sales and prices to reallocation. Quantitatively, brand reallocation reduces welfare. Optimal policies deviate substantially from the literature due to the complementarity between brand capital and productivity.

Key Words: Firm Dynamics, Productivity, Market Concentration, Product Innovation, Reallocation, Mergers & Acquisitions, Brands, Trademarks, Intangible Assets

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1 Introduction

Brands are an essential intangible asset for firms. The top 100 brands in the US economy were worth over $4 Trillion in 2021, and the relative value of brands to traditional capital has been growing over time (Bronnenberg et al., 2022).[1] We define a brand as a tradable form of customer capital; it is the means by which a firm accesses customers. Besides building customer capital gradually (Gourio and Rudanko, 2014), firms can also directly acquire customer capital through brand acquisition or *brand reallocation*.[2] This massive intangible asset class is not static – we find that 2% of trademarks are reallocated across firms each year.[3] As an intangible representing customer loyalty and attention, brands raise unique questions and insights in firm dynamics, concentration, and productivity. What are the macroeconomic implications of brand reallocation and brand capital on competition, aggregate productivity, and efficiency?

We develop a new dataset of *brands* with detail at the sales level and *trademarks*, defined as a transferable legal claim on brand ownership, to answer these questions. With this data, we develop three facts to motivate our discussion and provide a benchmark for the analysis in the paper.

- **Fact 1: Brands Matter.** Brands explain a significant amount of firm market share. Brands take time to build, but on their own explain 60% of firm-level sales variation.

- **Fact 2: Brand Reallocation Matters.** Brand reallocation contributes to a significant amount of across-firm changes in market share (more than 25%) and goes to larger firms, driving persistent leadership.

- **Fact 3: Productive and Strategic Reallocation.** When brands are reallocated across firms, sales and prices both increase, indicating potentially productive (sales $\uparrow$) and strategic (markup $\uparrow$) effects.

We put these facts at the center of a theoretical, empirical, and quantitative analysis of brand capital and brand reallocation. We focus on the nature of brand capital to amplify productive firms as well as deliver market power for both productive and unproductive firms. We use event studies and firm sales dynamics to quantify the theoretical model. We find that brand reallocation is *net inefficient*; on average, shutting down reallocation increases welfare by 1.4%. Once the dynamic joint allocation of brand capital are taken into consideration, classical size-dependent policies can be inefficient. These findings suggest alternative approaches to interpretations of productivity measurements, optimal policies in the presence of large firms, and modeling firm dynamics. We discuss the theoretical, empirical, and quantitative components in turn.

Our theoretical model opens with two competing firms that differ in their productivity, which is exogenous, and brand capital, which they can exchange or reallocate. Productivity evolves exogenously, with entering firms replacing technological followers. Our novelty is the introduction of brand capital, which evolves both exogenously and endogenously through brand reallocation. When firms transact brands, they can increase joint profits by sorting brand capital to the better firm and consolidating monopoly

[1] The estimated $4.14 Trillion represents 47% of the total value of Property, Plant, and Equipment at the same firms.

[2] In our language, brand capital is the tradable component of customer capital, as it is linked to a symbol that is separable from the firm.

[3] This phenomenon is fairly uniform across industries, as defined by NICE codes and NAICS classes, speaking to the broad prominence of brands and brand reallocation in the economy.

power. Thus, two incentives arise in brand reallocation: (1) a *productive* motive from sorting brand capital to the more productive firm, related to findings in M&A (David, 2020) and (2) a *strategic* motive as the larger firm is able to appropriate more profit from each unit of revenue (modeled following Atkeson and Burstein, 2008). Firms are only concerned with their own profits, which creates externalities in the marketplace: reducing active competitors increases profits and reduces the ability of customers to substitute away from the largest firm. In addition to the static distortion due to markups (Edmond et al., 2015), the externalities can also create a dynamic mismatch between firm productivity and brand capital. An unproductive firm could have a large market share and growth rate for an extended period due to its accumulated brand capital from the past, which persists through the acquisition of new brands. This persistent inefficiency is a unique mechanism when firms' intangible investments interact with variable markups.

Due to the countervailing productive and strategic motives of brand reallocation and their externalities, the welfare implications are ambiguous in the theoretical model. Uncovering the welfare implications requires empirical analysis. More specifically, it requires decomposing reallocation events into productive and strategic events. This leads us to center our empirical analysis on the role of brands in the US economy. Are brands and brand reallocation an important driver of market share dispersion? Is there evidence of productive and strategic motives in reallocation? To answer these questions, we construct a novel dataset of firms' brands in their trademark holdings, which we link to retail prices and sales. In line with our theory, trademarks represent a claim on customer recognition. The US Patent and Trademark Office (USPTO) defines a trademark as "any word, phrase, symbol, design, or a combination of these things that identifies your goods or services. It's *how customers recognize you in the marketplace* and distinguish you from your competitors." The USPTO records all federal trademarks that are registered, canceled, or reassigned. Through this dataset, we can locate brand reallocation events across firms via trademark reassignment. On average, 2% of trademarks are reallocated across firms per year. This rate is similar across all sectors, indicating that brand reallocation is a consistent feature of the economy.[4] While our empirical analysis is grounded in consumer packaged goods (CPG), the message on the importance of brand reallocation can be extended beyond CPG.

We use the linked trademark-brand dataset to study the joint evolution of firms and brands. We decompose the variance of market shares at the firm level and show how much of the variance in sales dispersion at large firms comes from brand reallocation. For firms with more than 1% market share in a given product group, 50% of the firms' variance in sales is due to reallocation. The role of reallocation is especially stark for the largest firms; without reallocation, the largest firms would be 10% smaller after a decade. We then focus on brand reallocation events. Relative to similar non-reallocated brands, we find that the average reallocated brand experiences an average of 43% increase in revenue and a 4% increase in price over the three years after reallocation. Interpreted through the lens of our theory, this suggests both a productive motive (sales $\uparrow$) and a strategic motive (price $\uparrow$). Leveraging variation at the retailer level, we find the price effect is stronger for retail chains where the buying firm has a larger market share, consistent with the assumption that a firm's markup increases in its market share, which has been found

[4] We discuss this further in Appendix C.1. There are 46 NICE codes that indicate the domain of the trademark. These NICE codes can be mapped to NAICS codes. See Kost et al. (2019) for a more detailed analysis on brand reallocation across NAICS categories.

in previous literature at the market level (e.g., Amiti et al., 2019) and consumer level (e.g., Afrouzi et al., 2023).

To quantify the welfare incidence of brand reallocation, we extend our simple model to a general equilibrium setting with richer features. We use our calibrated model to answer two questions regarding brand reallocation: how big is the mismatch between brand capital and productivity, and how does understanding the joint determination of customer base and productivity change the current policy remedies to market power? We find that brand reallocation is on net inefficient: without brand reallocation, markets are 1.4% more efficient.

We direct our attention to whether market leaders are efficient, as in the ideal market, brand capital is sorted to the most productive firm. In the stationary distribution, we find that the most productive firm is not the market leader in 42% of markets. Notably, not all of these markets experience inefficient reallocation. In 20% of markets, this inefficiency leads to reallocation from the more productive firm to the less productive firm. This inefficiency foreshadows an inefficiency that emerges in standard size-dependent policies, as the largest firm may not be the most productive.

Our quantitative model highlights the importance of understanding the determinants of firm market share as driven by brand capital and productivity independently. Most results in the literature (such as Boar and Midrigan, 2019 Berger et al., 2019, and Edmond et al., 2023) focus on markets where only productivity determines market share. These papers find that specific size-dependent policies (subsidizing large firms with high markups) restore efficiency in markets with market power. We recast the same set of policy instruments in the literature in our environment, where productivity and brand capital jointly determine market share and compare them to the planner's solution. In our environment, a size-dependent policy indeed corrects static distortions due to markups but can exacerbate the dynamic mismatch between brand capital and productivity. This exacerbation is because firms may be large due to customer accumulation, not productivity, and these subsidies promote more brand consolidation to those firms. This dynamic distortion can undo the welfare gains from correcting markups.

In our baseline calibration, the welfare loss due to the dynamic distortion amounts to 68% of the welfare gains that come from correcting markups. This distortion can even undo the welfare gains due to correcting markups. This is especially true in markets where goods are highly substitutable, as size-dependent policies lead to a 3% welfare loss compared to the decentralized equilibrium.

The remainder of this section reviews the literature, while the rest of the paper is structured as follows. Section 2 introduces the model framework and showcases the main mechanism in an analytical case. Section 3 introduces the data and empirical analysis of brands and firms jointly. Section 4 unites the theory with the data to uncover the parameters for quantitative analysis. Section 5 quantifies the welfare incidence of brand reallocation. Section 6 discusses policy implications. Section 7 discusses the robustness and extensions of our main results.

Related Literature. This paper builds on and contributes to several strands of the literature: the macroeconomics of M&A and technology transfers; the study of customer capital in macroeconomics; the study of firm dynamics, product dynamics and productivity; and the study of brands and branding.

The macroeconomic implications of mergers and acquisitions (M&A) and technology acquisitions have received rising interest. David (2020) studies the aggregate implications of M&A and finds that M&A increases overall efficiency in sorting productive assets to productive firms, an important channel for efficiency gains in our framework. Many other papers study the implications of technology and patent transfer in particular (such as Eaton and Kortum, 1996, Akcigit et al., 2016, and Shi and Hopenhayn, 2017). Whether this customer capital is sorted to efficient firms relates to studies on strategic versus efficient transactions in the market for firms and IP assets (Spearot, 2012, Abrams et al., 2019, Cunningham et al., 2021). Recent papers have focused on IP reallocation in a dynamic setting (Cavenaile et al., 2021 and Fons-Rosen et al., 2021). Bhandari et al. (2021) focus on the reallocation of indivisible capital amongst private businesses and the impact on optimal tax policy. Our paper is complementary to these set of papers but we differ in putting tradable customer access at the center of this discussion and highlight that variable markups can lead to dynamic mismatch in brand capital and productivity.

One reason for firm acquisitions is to acquire customers, an important ingredient in firm value and market share (Dinlersoz and Yorukoglu, 2012; Gourio and Rudanko, 2014). On a related note, Hottman et al. (2016) study multi-product firms and find that the "appeal" of products and firms, or residual customer demand, explains a large share of sales variation across firms. We build on this framework and study the dynamics of this appeal which highlights the role of brand reallocation. Argente et al. (2018, 2020b) and Jaravel (2018) explore how product creation and destruction are ubiquitous in product markets. Argente et al. (2021) and Einav et al. (2021) document that the expansion of product sales is primarily due to an expanding customer base. We provide a theory that studies different incentives of firms in building customer capital and link it to the data. We also connect to the literature in productivity and welfare that asks about the role of products (Bils and Klenow, 2001; Broda and Weinstein, 2006) the role of firm heterogeneity (Syverson, 2004a; Hsieh and Klenow, 2009), misallocation (Restuccia and Rogerson, 2017; David and Venkateswaran, 2019) and market power (Syverson, 2004b; Melitz and Ottaviano, 2008).

Productivity is a supply-side concept. Our goal is to develop insights on how productivity interfaces with demand-side factors; economists have recently noted demand plays a central role in understanding productivity and its measurement (Foster et al., 2008; Syverson, 2011, Sterk et al., 2021, and Michelacci et al., 2022). In standard models, large firms have better expertise or technology and produce more due to this supply-side advantage. However, there is growing evidence that demand or appeal plays a large role in the firm size distribution (Hottman et al., 2016). If a firm's higher market share comes from consolidating customer capital, it may generate a mismeasurement of productivity. In this paper, we analyze a case where larger firms may be less productive due to demand-side consolidation. We find that this is a pervasive feature of product markets.

Brands and brand reallocation also provide another avenue for studying the link between firm dynamics, market concentration, and markups (such as De Loecker and Eeckhout, 2018; De Loecker et al., 2020; Gutiérrez and Philippon, 2017; Eggertsson et al., 2018; Hall, 2018; Autor et al., 2020). Kehrig and Vincent (2018) find evidence of rising concentration and reduction in the labor share and connect this to previous marketing expenditures. Smith and Ocampo (2022) document the rise of market concentration, a significant force in product markets. This concentration can lead to markups. Boar and Midrigan (2019) link markups to inequality. Bornstein and Peter (2022) and Afrouzi et al. (2023) link markups to misallocation at the customer level.

We connect these discussions to firm dynamics and competition by building on the long literature of creative destruction (Aghion and Howitt, 1992, Aghion et al., 2001, Peters, 2020, and Liu et al., 2022), and augment this with literature on entry and firm development (Jovanovic, 1982; Hopenhayn, 1992). In our case, the competition over market share exhibits business stealing effects that firms ideally want to avoid. Some papers in this tradition focus on the links between factor or labor reallocation and growth (Acemoglu et al., 2018; Garcia-Macia et al., 2019), while we focus on tradable intangible capital. Jones and Williams (1998, 2000) study how markups and innovation interact to determine over- or under-investment in the creation of new products. Baslandze et al. (2023) study the introduction of new products empirically and theoretically. Edmond et al. (2015) focus on the markup channel, as large firms can leverage their large market share to charge high markups, a feature we explore in this paper. Amiti et al. (2019) find that strategic considerations occur in large firms' pricing decisions but not small firms' pricing decisions. To model this mechanism, our paper builds on Atkeson and Burstein (2008), who introduces an oligopolistic competition model with large multi-product firms where concentration and markups are jointly determined. In speaking to the role of tradable brand capital, we address the interaction of market concentration with intangible assets and customer acquisition (Bhandari and McGrattan, 2020), which shows up in advertising investments (Cavenaile and Roldan-Blanco, 2021; Greenwood et al., 2021) and can affect aggregate growth (Ignaszak and Sedlácek, 2022; Cavenaile et al., 2023). These papers connect more broadly to a set of papers that focus on the role of intangible forces in shaping modern markets, concentration, and growth (Haskel and Westlake, 2017, Crouzet and Eberly, 2019, Syverson, 2019, Akcigit and Ates, 2021, 2023, De Ridder, 2024).

Lastly, we bring insights from the literature on brands and branding to the macroeconomic debates on concentration, markups, and productivity. Brands have long been known to be an important source of firm values (e.g., Braithwaite, 1928 on brands, and Brown, 1953 on trademarks). Bain (1956) noted that "(t)he advantage to established sellers accruing from buyer preferences for their products as opposed to potential-entrant products is on the average larger and more frequent in occurrence at large values than any other barrier to entry." Nelson (1970) pointed out that market power is closely linked to customer attention. Theoretically, brands can generate persistent profits in markets with imperfect information (Schmalensee, 1978, Schmalensee, 1982, and Shapiro, 1983). The power of branding has been detailed empirically as consumer brand preferences are quite persistent (e.g., in Bronnenberg et al., 2009, 2012) and thus provide firms significant and tradable value (Tadelis, 1999). Trademarks serve as the central institution that links the property right to the brand (Economides, 1988). Recently, more work has developed insights on firm dynamics with trademarks (Dinlersoz et al., 2018, Heath and Mace, 2019, Castaldi, 2019 and Kost et al., 2019 stress the high degree of activity in the market for trademarks). Our paper builds on these ideas by linking brand capital to the market shares of firms and studying how brands can be reallocated across firms. Our approach adds tradability to classic work in industrial organization on advertising and market structure (Butters, 1977, Grossman and Shapiro, 1984, Sutton, 1991, and Stegeman, 1991).

2 A Theory of Brand Development and Reallocation

We introduce a theory of brands and brand reallocation that can be brought to Nielsen and USPTO data. We start with an equilibrium model of competition among oligopolistic firms with heterogeneous non-transferable productivity and transferable brand capital. We use this environment to explore a simple analytical case that highlights some themes of our model such as the inconclusive impact of brand reallocation on welfare and the path dependency of outcomes depending on the initial brand gap. In Section 4, we connect our predictions to the empirical evidence and quantify our general model. Section 6 studies the normative implications of the quantified model.

2.1 Environment

Time is continuous. There is a representative household, a set of product groups, and two duopolists within each product group. Firms in each group compete for household demand which they can obtain through brand capital and productivity. Dynamically, firms can exchange brand capital to either deliver greater allocative efficiency or obtain monopoly power.

Household, Firms, and Brands. The representative household endogenously supplies L_t units of labor and consumes branded products across a measure 1 of product groups. The product groups are indexed by $k \in [0, 1]$. The real consumption of the household, $\mathbf{C}_t$, is aggregated across product groups by a *constant elasticity of substitution* aggregator (hereafter, *CES*) in the following fashion,

$$\mathbf{C}_t = \left(\int_0^1 C_{kt}^{\frac{\theta-1}{\theta}} dk \right)^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{1}$$

$$C_{kt} = \left(\sum_{j=1,2} e^{\frac{1}{\sigma}(\alpha^D_{jkt} + \beta_{jkt})} c_{jkt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \sigma > \theta. \tag{2}$$

In Equation (1), C_{kt} is the consumption from product group k. Within each product group, there are two large firms (which we refer to as duopolists). The consumption at the group level, c_{jkt}, is another CES aggregator from the consumption of the products from each firm j. The elasticity across product groups is $\theta > 1$, and the elasticity within product groups is $\sigma > \theta$. This common assumption (e.g., Atkeson and Burstein, 2008) implies that large firms can gain market power as they internalize their impact on their own product group, which we will discuss when we characterize the pricing equilibrium.

Our model takes a similar structure to innovation models with duopolistic competition, such as Akcigit and Ates (2021) and Liu et al. (2022), but ours departs on the determinants of firm heterogeneity. The classic models of firm innovation focus on one dimension of heterogeneity, firm productivity. In our framework, not all firm advantage is due to firm productivity.

Our model separates the core productivity of firms, which is not transferable, and brand capital, which is transferable. These two components of our heterogeneity highlight two central and distinct inputs into firm value creation: the technology and the brand. A great technology cannot deliver to market without sufficient customer awareness. The following paragraphs define these components and their evolution starting with productivity and turning to tradable brand capital.

Productivity. Firms differ in their appeal productivity and labor productivity. Firm appeal, $\exp(\frac{1}{\sigma}\alpha^D_{jkt})$, captures the appeal of firms in their images to customers and distributional efficiency, as modeled in, for example, Hottman et al. (2016) and Argente et al. (2020c). Firms also differ in their labor productivity, $\exp(\frac{1}{\sigma-1}\alpha^L_{jkt})$. We refer to the summation of these two terms as *firm productivity* : $\alpha_{jkt} \equiv \alpha^D_{jkt} + \alpha^L_{jkt}$. The distinction between demand-side and supply-side productivity does not play a role in our theoretical results but maps into different empirical implications on price and revenue changes.

There is a *frontier* firm, denoted as firm 1, and *vintage* firm, denoted as firm 2. There is a net productivity gap between the two firms, $a_{kt} = \alpha_{1kt} - \alpha_{2kt}$. Productivity is dynamic and evolves according to a creative destruction process. With a Poisson arrival rate of γ, a new frontier firm enters and improves upon the old frontier firm with a step size a. We refer to γ as the creative destruction rate and a as the creative destruction step size. In this event, the old frontier firm becomes the new vintage firm, and the old vintage firm exits the market. This process is similar to endogenous growth models such as Akcigit and Ates (2023) and Peters (2020). In our baseline model, we take this creative destruction process as exogenous, as many other forces outside our paper's scope determine the innovation process. We provide an extension where we endogenize the innovation process in the Section 7.

Brand Capital. The core addition of our theory is the dynamics of brand capital. Firms differ in their brand capital β_{jkt}, which amplifies the connection of firms to their customers. We denote the gap of brand capital between the frontier and the vintage firm as $b_{kt} = \beta_{1kt} - \beta_{2kt}$.

Building on the theory of limited customer attention, we normalize the total brand capital in each product group to be $\sum_j e^{\beta_{jkt}} = 1$. This normalization takes a zero-sum view of branding.[5] The change of brand capital at the firm level does not directly create utility, but reallocates customer attention across firms, which can be efficient or inefficient depending on firm characteristics. This implies that brand capital and productivity at the firm level are complementary. Due to the limited attention in the aggregate, the economy faces a sorting problem between brand capital and productivity. When one firm gains brand capital another firm must lose brand capital, which makes the joint allocation of productivity and brand capital crucial for efficiency and aggregate productivity.

Upon entry, the entrant is a frontier firm in productivity, but starts with little brand capital, which is captured by an initial brand gap, $b_0 < 0$. A smaller b_0 can be interpreted as a smaller initial brand gap for new firms relative to the vintage firm.

The brand capital of firms can change endogenously due to brand reallocation, which occurs at rate λ.[6] This arrival rate of reallocation is a result of the joint decision of two firms. In a reallocation event, the relative brand capital between two firms changes by Δ. Reallocating brand capital is costly. Choosing an arrival rate of λ incurs a labor cost of $R(\lambda)$. $R(\lambda)$ is increasing, convex, differentiable, and $R(0) = 0$.

[5] In Pearce and Wu (2022), we focus on a model with brands in the utility. Given the primary focus on reallocation, this discussion has less relevance for this paper.

[6] This is the main mechanism that we focus on in this paper, though Section 7 extends our model to the study of endogenous advertisement which builds brand capital directly.

We define this endogenous arrival rate as the reallocation intensity. Firms split the surplus of the value of reallocation using Nash bargaining, where the frontier firm has a bargaining power of ϕ and the vintage firm has a bargaining power of $1-\phi$. The assumption of Nash bargaining and contractable reallocation cost implies that the reallocation decision is always bilaterally efficient. This bilateral efficiency also implies the bargaining power per se is not important for the baseline model as reallocation is the only endogenous choice, which we show next in the value function. Firms' brand capital also moves for idiosyncratic reasons, which we capture by a Brownian motion of the grand gap with volatility ν.

Aggregation. All activities in the economy use labor as the input. In the baseline model, labor is allocated to production and reallocation activity. The labor resource constraint of the economy thus requires:

$$\mathbf{L}_t = \int_0^1 \left(\sum_{j=1,2} \frac{c_{jkt}}{e^{\frac{1}{\sigma-1}\alpha^L_{jkt}}} \right) dk + \int_0^1 R(\lambda_{kt}) dk. \tag{3}$$

The rest of the household's problem is standard. We assume the household has a flow utility of $\log \mathbf{C}_t - L_t$.[7] Households spend on consumption and hold assets $\mathbf{A}_t$ that evolve over time. The household can borrow and save in a representative portfolio of all firms, such that the aggregate profit Π_t is rebated to the household as a dividend. We define ρ to be the discount rate, r_t to be the interest rate, and normalize the wage to be 1. We write the household's problem as

$$\max_{c_{jkt}, \mathbf{L}_t} \int_0^\infty e^{-\rho t} \left(\log \mathbf{C}_t - \mathbf{L}_t \right) dt,$$

s.t.

$$\dot{\mathbf{A}}_t = r_t \mathbf{A}_t + \mathbf{L}_t + \mathbf{\Pi}_t - \int_0^1 \sum_{j=1,2} p_{jkt} c_{jkt} dk$$

Oligopolistic Competition. The structure of competition has important implications for brand reallocation across firms. We assume that the duopolists take as given the aggregate price index across all product groups, $\mathbf{P}$, yet are large enough to internalize their impact on their own product group (as in Atkeson and Burstein, 2008).[8] As a result, they charge a variable markup in the equilibrium, which is a function of their market shares, $\mu(s_j) = \frac{\epsilon(s_j)}{\epsilon(s_j)-1}$, and thus the ability of consumers to substitute away from the firm, or perceived elasticity, $\epsilon(s)$. We assume firms engage in Cournot competition, which implies that the perceived elasticity, as a function of their market share s, is $\epsilon(s) = \left(\frac{1}{\sigma}(1-s) + s \right)^{-1}$.

2.2 Characterization

We now characterize the equilibrium, moving from the household to the firms' pricing decisions, to the dynamic reallocation decisions. We then define equilibrium and discuss welfare.

[7] The assumption of linear labor disutility simplifies the characterization of equilibrium by assuming away the crowding out of reallocation activity to production activity. This assumption is not essential in our quantitative analysis.

[8] This can be micro-founded as a multi-product firm in a given category, as in Pearce and Wu (2022). This captures the fact that firms in given product groups provide many products and brands within the same group.

Household Decision and Pricing Equilibrium. The optimal choice of consumption from the household leads to a standard constant elasticity of substitution (CES) demand curve for products held by firm j,

$$c_{jkt}(p) = \exp(\alpha^D_{jkt} + \beta_{jkt}) \left(\frac{p}{P_{jkt}}\right)^{-\sigma} \frac{P_{jkt}^{1-\theta}}{\mathbf{P}^{1-\theta}} \frac{\mathbf{PC}}{P_{jkt}}, \tag{4}$$

and consumption-leisure tradeoff implies $\mathbf{PC} = 1$. Household demand for firm j's products is a function of the relative price of goods (p), the elasticity of substitution (σ) and $\alpha_{jkt} + \beta_{jkt}$, which we refer to as the *effective* productivity of the firm. This notion of effective productivity is the core productivity of the firm, α_{jkt}, scaled by the brand capital it holds, β_{jkt}. A firm with large productivity but no brand capital $\beta_{jkt} \to -\infty$ will have negligible market share. This highlights the the complementarity between brand capital and firm productivity.

There are only two firms in each group and only their relative effective productivity gap, $a + b$ for the frontier firm, is what matters for the determination of group-level markup and market shares. The market share of the frontier firm, s, is thus determined by the effective productivity and markups μ such that,

$$\frac{s}{1-s} = e^{a+b} \frac{\mu(s)^{1-\sigma}}{\mu(1-s)^{1-\sigma}}. \tag{5}$$

We denote the solution to Equation (5) as $s(b)$. $s(b)$ is the frontier firm's within-group market share when its brand gap with the vintage firm is b. When the duopolists trade in brand capital, they are interested in maximizing their joint profit,

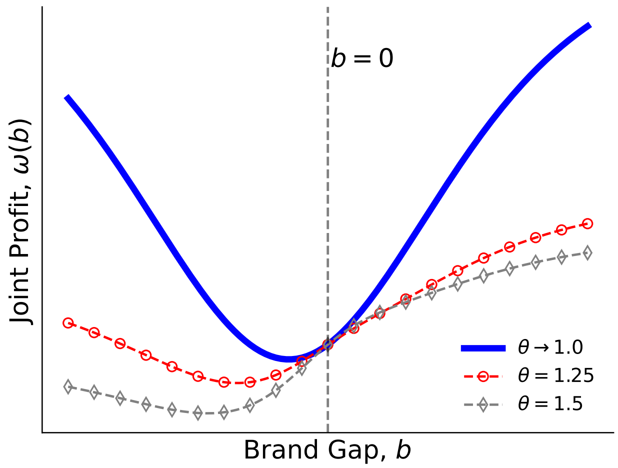
$$\omega(b) = \underbrace{S(b)}_{\text{Across Group}} \times \underbrace{\left(s(b)\frac{1}{\epsilon(s(b))} + (1-s(b))\frac{1}{\epsilon(1-s(b))}\right)}_{\text{Within Group}}. \tag{6}$$

In Equation (6), the joint profit of the duopolists depends on both the total market share $S(b)$ of the product group and the within-group distribution of market shares. We visualize this in Figure 1, which plots the joint profit of the duopolists against the brand gap between the firms for a fixed productivity gap.

The size of the product group relative to the aggregate depends on two forces. First, it depends on the group-level productivity $Z(b) = \left(\frac{e^{a+b}+1}{e^b+1}\right)^{\frac{1}{\sigma-1}}$, which is increasing in the brand gap. Due to the complementarity between brand capital and productivity, a higher brand gap means the frontier firm holds more brand capital. This increases the frontier firm's effective productivity more than it decreases the vintage firm's effective productivity, which leads to higher group-level productivity. However, the total output also depends on the group-level markup, which is a function of the frontier (μ_1) and vintage (μ_2) as follows, $M(b) = \left(\frac{e^{a+b}\mu_1(b)^{1-\sigma}+\mu_2(b)^{1-\sigma}}{e^{a+b}+1}\right)^{\frac{1}{1-\sigma}}$. A more consolidated market (high $|b|$) means higher markups, regardless of whether the brand capital is sorted to the frontier or vintage firm.

As the creative destruction step size a is fixed, the only endogenous variable that affects the group-level productivity and markup is the brand gap b. The response of profits to brand gap b for given cross-group substitution elasticities θ, is given in Figure 1.

Figure 1: Joint Profit



Notes: This figure plots the joint profit of duopolists given a brand gap for different values of the cross-group elasticity θ, with $\sigma = 4.0$. All points normalized the aggregate expenditure of the household to be 1.

The figure indicates the relationship between the returns to the firm and the brand gap. For all values of cross-group elasticity θ, the joint profit is increasing in the brand gap when $b > -a$. This shows the return for firms of sorting brand capital to the more productive firm as well as the higher markups that come from consolidation. When the cross-group elasticity is low ($\theta \to 1$), firms also benefit from consolidating to the less efficient firm due to the markup effect of consolidation (blue line when $b << 0$).

The returns to consolidation are generally convex, which generates the increasing returns when $|b|$ is high. A special case is useful to highlight the static equilibrium outcomes. If $\theta = 1$, we can solve this joint profit function in closed form:

$$\omega(a+b) = \frac{1}{\sigma} + \left(1 - \frac{1}{\sigma}\right) \frac{1 + 2e^{(a+b)/\sigma}}{(1 + e^{(a+b)/\sigma})^2}.$$

In this case, the joint profit function is increasing in the absolute value of b, which drives convex returns to consolidation. This static return to reallocation informs the dynamic decisions, which we turn to next.

Dynamic Decisions. In our baseline model, the only dynamic decision is the reallocation of brand capital.[9] We denote the discounted value of the frontier firm as $V(b)$ and the discounted value of the vintage firm as $v(b)$. For convenience of characterizing the reallocation decisions, we denote their joint surplus as $\Omega(b) = V(b) + v(b)$. The frontier firm's discounted value is the solution to the following Hamiltonian-

[9] The baseline model in this paper only focuses on brand reallocation. As noted by other papers, brand capital is also endogenous to choices at the firm level absent reallocation; these rich dynamics are explored in extensions with endogenous advertisement and innovation in Section 7 and in Pearce and Wu (2022).

Jacobian-Bellman Equation (HJB):

$$\rho V(b) = \Pi(b) + \phi\bar{\Omega}(b) + \gamma(v(b_0) - V(b)) + \frac{\nu^2}{2}V''(b). \tag{7}$$

The frontier firm receives profit $\Pi(b)$ every instant. Firms endogenously choose an arrival rate λ that delivers the chance to reallocate brand capital across the two firms. When this occurs, the joint return $\bar{\Omega}(b)$ is the maximized gains from trade, from which the frontier firm receives a share ϕ and vintage receives share $(1-\phi)$. Creative destruction occurs at rate γ, and a new duopolist enters and becomes the frontier firm, with the old frontier firm becoming the vintage firm. The relative brand gap between the frontier and vintage firm changes with Brownian motion of volatility ν. The discounted value for the vintage firm is similar to Equation (7), except that the vintage firm exits and receives zero when the creative destruction happens,

$$\rho v(b) = \pi(b) + (1-\phi)\bar{\Omega}(b) + \gamma(0 - v(b)) + \frac{\nu^2}{2}v''(b). \tag{8}$$

The joint surplus $\Omega(b)$ is defined as the sum of the discounted values of the frontier and the vintage firm $\Omega(b) \equiv V(b) + v(b)$. From Equation (7) and Equation (8), we reach the following equation for $\Omega(b)$:

$$\rho\Omega(b) = \omega(b) + \bar{\Omega}(b) + \gamma(v(b_0) - \Omega(b)). \tag{9}$$

The optimized value from reallocation is given by:

$$\bar{\Omega}(b) \equiv \max_{\lambda} \lambda \max\left\{\Omega(b+\Delta) - \Omega(b), \Omega(b-\Delta) - \Omega(b)\right\} - R(\lambda).$$

There are two pieces to this optimal reallocation decision. First, the firms jointly decide how much effort to put into reallocation, which changes λ. When the opportunity arises, they can decide whether the brand capital is reallocated towards the frontier or the vintage firm. We denote $\lambda^+(b)$ and $\lambda^-(b)$ as the optimal intensity of reallocation, conditional on the firms reallocating brand capital towards the frontier firm and vintage firm, respectively.

To characterize the equilibrium, it is sufficient to focus on Equation (9) and the joint surplus. This simplification occurs because the only dynamic choice is reallocation. Another implication is that the rent splitting between the selling and the buying firms is irrelevant in our baseline model, since the bargaining power ϕ does not show up in Equation (9). In the extension with endogenous advertising, these properties no longer hold.

Equilibrium Definition. In Equation (7), for b_0, the distribution of product groups with brand capital b evolves according to:

$$\dot{g}(b) = -(\lambda(b) + \gamma)g(b) + \lambda^+(b-\Delta)g(b-\Delta) + \lambda^-(b+\Delta)g(b+\Delta) + \frac{\nu^2}{2}g''(b), \tag{10}$$

with a restriction $\int_b g(b)db = 1$. Together, with the condition that limit $\lim_{b\to\infty} g(b) = \lim_{b\to-\infty} g(b) = 0$, this equation uniquely pin down the distribution of product groups in their brand gap b. There are three forces driving the dynamics of brand gaps. The reallocation rate, which is proportional to λ; the creative destruction rate, which is proportional to γ; and the exogenous fluctuations in brand capital, which are proportional to $\frac{\nu^2}{2}$.

We are now ready to formally define a steady-state equilibrium. This definition can be extended to incorporate transitional paths, which we discuss in the Appendix.

Definition 1 (Steady-state Equilibrium) *A steady-state equilibrium is a collection of static pricing equilibrium decisions $\omega(b)$, dynamic decisions $\{\Omega(b), \lambda^+(b), \lambda^-(b)\}$, and distribution $g(b)$ such that:*

1. *(static equilibrium): $\omega(b)$ solves the static pricing equilibrium ;*
2. *(dynamic optimal): $\{\Omega(b), \lambda^+(b), \lambda^-(b)\}$ solve Equation (9);*
3. *(stationarity): $g(b)$ solves Equation (10).*

The steady-state equilibrium can be computed recursively from their respective equations.

Aggregation and Welfare. We present a heuristic discussion of overall welfare which we expand on in Section 6. The household's utility depends on both the dispersion of markups and on whether brand capital is allocated toward the productive firm. We write out the consumption from a product group with productivity gap a and brand gap b.

Lemma 1 (Aggregation) *In a steady-state equilibrium, the discounted utility of the household is given by:*

$$\mathbf{W} = \frac{1}{\rho}\Bigg(\underbrace{\log\frac{\mathbf{Z}}{\mathbf{M}}}_{\textit{consumption}} - \underbrace{\frac{1}{\mathbf{M}}}_{\textit{production labor}} - \underbrace{\mathbf{R}}_{\textit{reallocation labor}} \Bigg),$$

where

1. $\mathbf{Z}$ *is the aggregate labor productivity*

$$\mathbf{Z} = \left[\int_b Z(b)^{\theta-1} g(b)db\right]^{\frac{1}{\theta-1}}, \quad Z(b) = \left(\frac{e^{a+b}+1}{e^b+1}\right)^{\frac{1}{\sigma-1}};$$

2. $\mathbf{M}$ *is the aggregate markup*

$$\mathbf{M} = \left[\frac{\int_b Z(b)^{\theta-1} M(b)^{1-\theta} g(b)db}{\int_b Z(b)^{\theta-1} g(b)db}\right]^{\frac{1}{1-\theta}}, \quad M(b) = \left(\frac{e^{a+b}\mu_1(b)^{1-\sigma} + \mu_2(b)^{1-\sigma}}{e^{a+b}+1}\right)^{\frac{1}{1-\sigma}};$$

3. $\mathbf{R}$ *is the aggregate labor cost in reallocation*

$$\mathbf{R} = \int_b R(\lambda(b))g(b)db.$$

Returning to firms' reallocation decisions, there are two externalities created by firms to the representative household. First, the dispersion of markups between firms creates a misallocation of labor. Firms' choose markups to maximize individual profit and not overall welfare. This misallocation reduces the productivity of labor. Second, the firms do not fully internalize the benefit of matching transferable brand capital towards more productive firms. Firms may stick with mismatch if it builds more monopoly power. The second externality is a novel insight from our paper. This equilibrium is not necessarily efficient, which creates room for policy interventions.

We wait to discuss optimal policies until after we have set up the planner's solution in Section 6. However, the analysis so far hints at the main role of policy. A policy that aims to improve efficiency should induce firms to sort brand capital in the right direction and reduce markups.

2.3 Analytical Case: Welfare Incidence of Brand Reallocation

In this section, we consider a simple case to highlight the role of brand reallocation on welfare and the sources of distortions. We proceed by characterizing the endogenous objects analytically. We make two simplifying assumptions for the analytical case: (1) we assume that the arrival of reallocation is constant at $\bar{\lambda}$, (2) we assume the size of each product group is fixed ($\theta = 1$), and (3) we assume away the brand exogenous volatility ($\nu = 0$). One way to micro-found the constant reallocation size is to assume the following cost function:

$$R(x) = \begin{cases} 0, & \text{if } x \leq \bar{\lambda} \\ \bar{R}, & \text{if } x > \bar{\lambda}, \end{cases} \tag{11}$$

where $\bar{R}$ is a very large number. Under these conditions, the following lemma holds:

Lemma 2 *Brands reallocate towards the frontier firm for any $b > -a$, and towards the vintage firm for any $b < -a$.*

Proof. See Appendix A.2. □

When $b > -a$, brands reallocate towards the more efficient firm. We call this *efficient reallocation*. When $b < -a$, brands reallocate towards the less efficient firm. We call this *strategic reallocation*. Why would firms reallocate the transferable asset towards inefficient use, when the firm productivity is a complement to the brand? Bigger firms are able to charge a higher markup, and thus an additional unit of brand capital is worth more to them. Large firms, for instance, have a larger incentive to control a brand in their market than a smaller firm with the same productivity. This mechanism can generate *persistent* mismatch between productivity and brand capital.

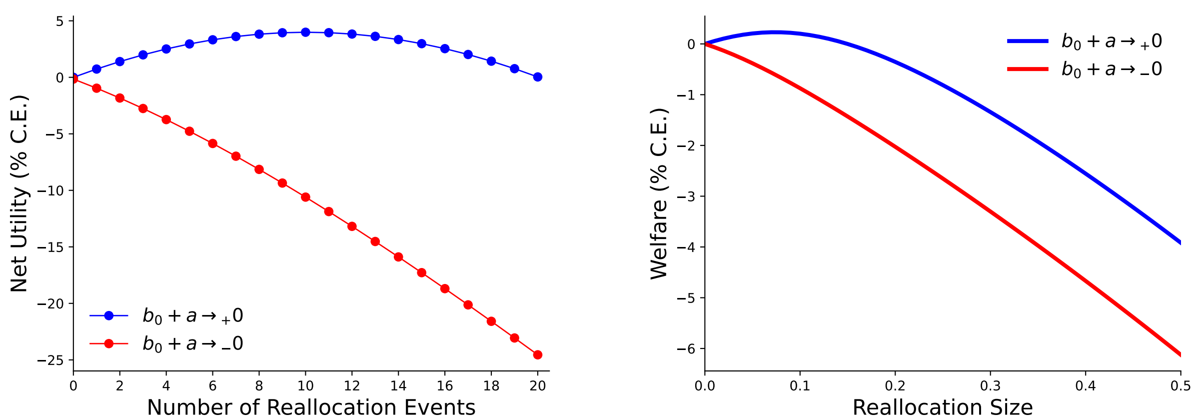
Lemma 3 *Assume $\nu = 0$ and the functional form for reallocation cost comes from Equation (11). The distribution of product groups with respect to b has a probability mass function at $\{b_i\}_{i=0}^{\infty}$ with probability $g_i = \frac{\bar{\lambda}}{\gamma}\left(\frac{\bar{\lambda}}{\bar{\lambda}+\gamma}\right)^{i-1}$, where $b_i = b_0 + i\Delta$ if $b_0 > -a$ and $b_i = b_0 - i\Delta$ if $b_0 < -a$.*

Proof. See Appendix A.3. □

In this analytical case, the variation across product groups comes primarily from how many times a group has experienced a brand reallocation opportunity before the next creative destruction. Across product groups, this delivers a geometrically distributed probability mass function of brand gaps. As a result, the model generates a form of path dependence. The starting brand gap b_0 is crucial for determining the direction of reallocation. When the initial brand gap is tilted too strongly to the vintage firm ($b_0 < -a$), brands are mismatched with productivity. When the initial brand gap is large enough, ($b_0 > -a$), brands are always matched with productivity.

Figure 2 presents two graphs that summarize the main message of the analytical dynamics. We focus on the relationship between welfare and the number of reallocation rates (panel A) and size of reallocation (panel B). We focus on two states of the world: the effective productivity is tilted towards the frontier firm ($b_0 + a \to 0_+$), and the effective productivity is tilted to the vintage firm ($b_0 + a \to 0_-$). Figure 2a reports the relationship between the number of reallocation opportunities and total welfare (net utility) in the economy in these two cases. Figure 2b reports the relationship between the amount possible to reallocate in one event (e.g., determined by policy or ownership frictions) and overall welfare in these two cases.

Figure 2: Analytical Case — Welfare and Reallocation



(a) Number of Reallocation Events

(b) Reallocation Size per Event

Notes: Analytical cases plots the utility against count of reallocation events (Figure 2a) and size of reallocation (Figure 2b). Panel (a) shows that when initial brand capital is slightly above the productivity gap (blue line), reallocation events tilt to productive outcomes, relative to unproductive (red line). Panel (b) shows the same for reallocation step size.

Figure 2a illustrates the role of path dependency on welfare: small deviations in initial brand capital are essential for brand capital flows and welfare. This is a core message of the analytical case. When brand capital is reallocated to the more productive firm, it increases welfare initially. This comes from a better match between brand capital and firm productivity. However, enough reallocation to the more productive firm leads to monopoly power and lowers consumer welfare, which delivers the non-monotonicity in the blue curve. The red curve, on the other hand, indicates the monotonic decline in welfare when strategic reallocation dominates. In this case, large firms hold brand capital in a persistent sense leading to long-run markup inefficiency and mismatch inefficiency.

Figure 2b fixes other parameters and varies the step size of reallocation, while keeping the stationary distribution of the number of events the same. We ask how does varying reallocation size affect the aggregate welfare? We find that when reallocation size increases beyond a certain level, firms can extract more profit margin from the market. This is especially negative for welfare when the initial state is tilted to the inefficient incumbent. Both of these cases shows the importance of understanding initial conditions, creative destruction, and the interaction of brand capital and productivity for welfare and optimal policy.

Market outcomes of mismatched and matched markets are observationally equivalent if we only observe the market shares. However, the economic and welfare implications are starkly different, which we explore in Figure 2 and Section 4. The non-monotonicities in this model immediately suggest a need for quantification to understand the qualitative and quantitative role of policy. The simple model enables us to link firms' activities in both sales to consumers and in trading brand capital to aggregate outcomes in the macroeconomy. We will expand on this stylized model in Section 4 to embed more empirical realism, but the main idea of linking firm productivity and brand capital to aggregate outcomes will remain central.

3 Data and Empirical Analysis

Our theory treats brand capital and core firm productivity as the joint determinants of market share. Empirically, there is a rich literature on firm productivity measurement on the supply side. The literature has been mostly silent on brand capital. This section introduces a crosswalk between firms' trademark holdings and brand-level data on prices, customers, and revenues to provide a foundation for measuring brand capital and its interaction with fundamental firm characteristics.

We start by discussing the data construction, as we link USPTO trademark data to information at the retail level on firm prices and sales. We then use the constructed dataset to develop core facts on brands and brand reallocation. The event studies, where brand ownership is reallocated across firms, provide a key ingredient in the decomposition of a firm and brand effect on market share. We bring the data directly to the model in the quantitative analysis in Section 4.

3.1 Data Construction

The first unique data contribution of this paper is to merge US Patent and Trademark Office (USPTO) trademark data with RMS Nielsen scanner data. The pairing provides details on brand history, including the prices, sales, and age of each brand, and firm-specific features, such as firm sales revenue and brand holdings. The merged dataset admits exploration of the mechanics of brand introduction, brand development, and brand reallocation.

USPTO trademark data provide a unique and comprehensive insight into the distribution and history of brands across firms. Individuals or firms apply for trademarks when they want legal protection for their brand capital. The USPTO defines a trademark as "any word, phrase, symbol, design, or a combination of these things that identifies your goods or services. It's how customers recognize you in the marketplace and distinguish you from your competitors." More firms participate in trademarking than patenting, and there are more trademarks than patents reallocated across firms each year. Within reallocation, we restrict our attention to reassignments and mergers.[10]

To enable the study of price and sales data, we employ detailed bar-code level data from Kilts-Nielsen Retail Measurement Services Data from the University of Chicago Booth School of Business. The data are large and comprehensive in the consumer product space. This dataset delivers significant coverage for products, brands, and firms, which we detail in Appendix B (see Argente et al., 2020b for more detail on this merge). One point of departure from the literature is our focus on brands rather than products. A product contains a specific 12-digit identifier, which may contain slight product variations under a broader brand (e.g., size differences, new editions, seasonal variations). Since our interest is primarily linked to the customer association with the product, we focus on a higher level of aggregation. This also simplifies the merge with USPTO trademark data.

We perform the merge by focusing on the brand and the firm as a pair. We employ a fuzzy merge to connect brand names in RMS Nielsen scanner data to USPTO trademark data. This merge is the first we know of that links USPTO *trademark* data to Nielsen scanner data; Argente et al. (2020a) link USPTO *patent* data to RMS Nielsen data. Compared to their merge, more firms and brands are matched in our sample, likely due to the different nature of patenting and trademarking. In particular, more firms trademark than patent, and all products in a brand umbrella can be identified as long as the trademarked brand name matches the brand name on the product in the store. We expand on the details of our merge in Appendix B. Table 1 details the merge on the trademark side and the RMS Nielsen side.

Table 1: Summary Statistics on Trademark–Nielsen Merge

	Unique Count	Years Active	Share Match (%)
USPTO Trademark Data			
Trademarks	5.36M	1870–2020	1.9%
Firms	371,021	1870–2020	15%
Canceled Trademarks	2.12M	1970–2020	
Transactions	915,076	1970–2020	
RMS Nielsen Scanner Data			
Products × Group	1.64M	2006–2018	
Brands × Group	82,525	2006–2018	57%
Firms	23,232	2006–2018	54%
Brand × sales		2006–2018	82%

Notes: Summary statistics on share of merge brands in both datasets. Source: USPTO Trademark Data and RMS Nielsen Scanner Data.

We stress a few points from Table 1. First, we capture 57% of the brands in the unweighted merge. When we merge brands weighted by sales, we capture a larger share within the Nielsen data, 82%, due to brands with more sales being more likely to be trademarked. Some small firms may choose not to protect their intellectual property via legal means. Second, many trademarks are not associated with consumer packaged goods, so a smaller share of trademarks are merged. Third, multiple brands are associated with a single firm, in line with the model; furthermore, multiple UPC products are connected to a single brand. On average, we observe 9 unique UPC products per brand. This connects to our framework where the brand is a capital good at the firm that provides a family of products an umbrella by which they access the customer.

[10]There are multiple types of transactions, which we discuss in detail in Appendix B.

3.2 Empirics of Brands and Brand Reallocation

We focus on the role of brands as transferable customer capital and use this framework to study the evolution of market share and the fixed and transferable components of firms. We start by breaking down the contribution of brand creation, maturity, and reallocation to firm growth and decline. We then study the effects of brand reallocation across firms on prices and sales of the focal products. This exercise provides insights into both the outcomes of brand reallocation and enables the decomposition of the brand and firm components as the main drivers of market share.

We start our empirical analysis with a study of brands mirroring the main features of the model. Our focus is primarily on the persistence of large firms, brand capital, and reallocation. One theoretical mechanism we propose is the potential mismatch between productivity and brand capital. Large firms may accumulate brand capital and corner the market rather than sell their brand capital to more productive firms. Brand reallocation thus enables this large unproductive incumbent to persist. On the other hand, brand reallocation can also generate efficiency gains by matching productive firms with customer access. With our framework, we are able to identify and measure this mechanism in the data.

We find that brand reallocation on its own drives significant market share dispersion, especially for large firms. We then study brand reallocation events, are they productive (to the better firm), strategic (driving up markups), or both? On average, brand reallocation leads to increases in sales (expanding customer base, an efficient outcome) and an increase in prices (an increase in the implied markup, an inefficient outcome). These reallocation events mask rich heterogeneity that we discuss briefly in this section and discuss in the quantification in Section 4.

Firm-level characteristics are the most common framework in the literature for explaining concentration and market share (Edmond et al., 2015; Akcigit and Ates, 2021). Our ability to observe brands move across firms provides insights into how much market share outcomes come from the firm versus the brand level. In the M&A literature, there is often missing data due to the inability to identify revenue in the acquired firm after the event. One can observe the technology transfer with patent data (e.g., Akcigit et al., 2016) but no outcomes directly linked to revenue, prices, or market share. In our event studies, we are able to continue to track brands throughout their life cycle with the corresponding revenue attached to them. Thus, tracking brands in event studies provides a unique way to measure the firm and brand-level effects.

Brands and Market Share. Large firms in consumer packaged goods hold persistent leadership and many brands. We define a market *leader* as the firm in a given product group with the largest market share, either within a year or over our entire sample. We define a market *follower* as the second-largest firm defined in the same way. Table 2 illustrates the persistence of leadership, the role of multiple brands in leadership, and the market share of leaders.

The odds that a leader retains leadership year-over-year is 88% (92% sales-weighted). Including the follower, the top two firms have a 97% chance of being the market leader in the next period. Furthermore, leading firms hold many brands. On average, a leading firm has 24 brands compared to the median firm's one brand. This puts the top two firms at the center of an analysis of market power and dynamics.

Table 2: Brands and Leadership

	(1)	(2)	(3)	(4)	(5)	(6)
		—*No Weights*—			—*Group Sales Weights*—	
	Leader	Follower	Other	Leader	Follower	Other
Leader Next Year	0.877	0.064	0.000	0.923	0.046	0.000
Market Share	0.326	0.164	0.001	0.299	0.159	0.001
Num. Brands	25	15	2	45	27	2
Exit Rate	0.015	0.023	0.113	0.013	0.015	0.108
Observations	1376	1372	534489	1376	1372	534489

Notes: Each row performs a variance decomposition on firm total sales that can be attribute to new brands, growth within brands held by the firm, and reallocation of brand ownership. Source: USPTO/RMS Nielsen, sample period: 2007-2017.

Leaders have around 30% market share, with the top two firms holding almost 50% market share, even when outnumbered by other firms 500-to-1. In Appendix C.3, we identify "mover" brands and build a connected set based on firm size following Bonhomme et al. (2019). We use this set to identify how much of market share can be explained by fixed firm characteristics versus the amount and quality of a firm's brands. We find that brands independently explain 60% of a firm's market share, with the firm fixed effect only explaining 20%, and the covariance explaining the rest. This suggests that brands are an essential asset for a firm's market share.

Reallocation and Persistent Leadership. In building market share, a classic tradeoff for firms is the choice between internal development and acquisition. Empirically, we bring this framework to the realm of brands.[11] This work builds on the is a growing body of evidence that the development of customer base is at the center of firm growth and markups (as noted by Einav et al., 2021 and Afrouzi et al., 2023).

In the following empirical exercise, we decompose the driving forces behind changes in firm market share in terms of brands. Embedding the classic tradeoff in the theory of the firm into branding, we focus on the relative roles of brand creation (new brands), brand incumbent growth (changes in the share of old brands), and brand reallocation across firms. We link these three components to the changes in market share within a given product group (e.g., "SOFT DRINKS"). We ask how much the variance of firm market share growth is driven by these three forces. For brand creation, we include brand introduction (the first year of a brand) and brand death (removal of brands from the market). For brand maturity, we include all growth and decay from brands held by their parent firm in two consecutive periods. For brand reallocation, we focus on the net transacted brands in terms of market share. For instance, if a firm sells a brand this counts negatively in terms of the market share lost, whereas if a firm acquires a brand, it counts positively in terms of market share gained.

We focus on a variance decomposition of the change in firm market share due to these three forces. Table 3 reports the total variance (first column) and the share explained by the three components above. This table presents the variance unweighted, with Appendix C.1 providing the weights by firm average size.

There are multiple takeaways worth stressing from Table 3.[12] When there are no firm cutoffs for size

[11] Appendix C.1 documents the broad prominence of reallocation for large firms and the nature of this reallocation, which is mostly from reassignment across firms.

[12] We leave out the covariance terms in this table for parsimony. Overall, covariances explain 5% or less of overall variance, indicating most of the variance in market share comes from the three sources of market share growth and decay.

Table 3: Variance Decomposition of Firm Market Share Growth, Weighted by Group Size

	(1) Total Variance	(2) New Brands	(3) Incumbent Growth	(4) **Brand Reallocation**
All Firms	0.72	6%	86%	3%
Firms >0.01% Share	0.24	7%	81%	10%
Firms >0.1% Share	0.14	7%	72%	21%
Firms >1% Share	0.07	5%	44%	**50%**
Firms >5% Share	0.04	1%	19%	78%

Notes: Each row performs a variance decomposition on firm total sales that can be attribute to new brands, growth within brands held by the firm, and reallocation of brand ownership. Source: USPTO/RMS Nielsen, sample period: 2007-2017.

the total variance of firm growth is large, due to the presence of many small firms, which are known to have higher variance than large firms. The share of firm growth variance driven by reallocation is small, as most of the variation in firm growth comes from incumbent growth.[13] When we increase the cutoff to 0.01% of the total market share in a given product group, the role of reallocation starts to become a major driver of market share, more important than new brand creation. For firms larger than 1% of the market, reallocation plays a more significant role than both creation and maturity, indicating its central role in the market share growth of large firms.

To understand the evolution of firm market share and the role of reallocation, Figure 3 plots the evolution of the average leader's share, average follower's share, and an average other firm's share over time.[14] We normalize the values to be comparable in the initial period. For the top two firms, we plot the evolution of their log market share (solid lines) and the counterfactual market share in a market without brand reallocation (dotted lines). We report this evolution over a 10-year period.

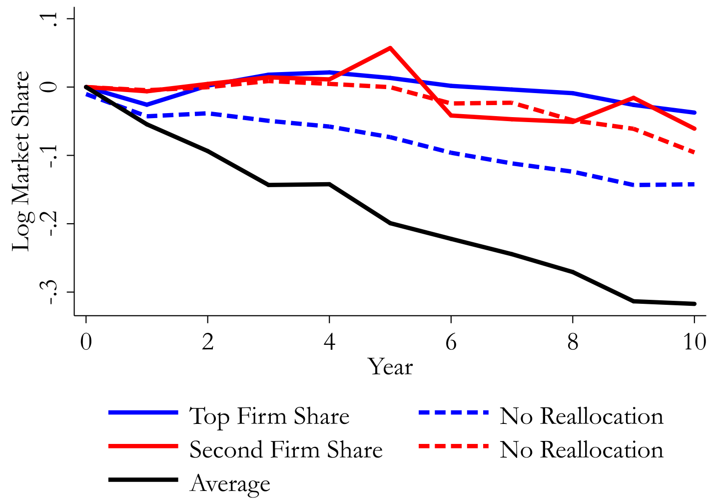
We highlight a couple facts from Figure 3. First, market share leaders are more persistent in their shares than the median firm. The gap between the top two firms and the average firm grows over time. After 10 years, both the leader and follower firms show a similar market share to the initial period, while the average firm has 30% less market share.

Second, the reallocation of brand capital explains a significant share of this persistence, in particular for the top firm. Without brand reallocation to the leader, the change in the gap of the log between the leader and the average firm would shrink by almost 40%. In contrast, the second firm relies more than the leader on its existing brands and the creation of new brands. While the second firm still has some gap at the end of the period, the gap is much smaller than the leading firm (around 2% difference versus 12% difference). Figure 3 indicates that acquiring brands is an important strategy for large firms to maintain their advantages with respect to small firms and a driver of the persistence of market share of leading firms.

Strategic and Productive Reallocation. We focus on two different outcome variables in our event studies: revenues and prices. We compare the reallocated brands to a group of similar brands within the same product group around the time of the reallocation event. We match on pre-event sales and product group categories and year, applying weights to generate a synthetic control, as discussed in Blackwell et al.

[13] In Appendix C.2, we focus on the nature of brand maturity and incumbent growth at the brand and firm level.
[14] We define leader as the firm with the highest average market share in a group from 2006-2018.

Figure 3: The Evolution of Market Share



Notes: Top two firms determined by average market share, logged and initialized at period 0. The blue line represents the leader and red line represents the follower. The dotted lines remove all net inflows/outflows from each firm contributing to market share. Shares are defined within each product group and weighted by product group size in sales. Source: USPTO/RMS Nielsen.

(2009). These similar brands will be comparable brands that did not experience a reallocation event. We then estimate the following regression,

$$\log y_{ikt} = \sum_{\tau=-3}^{3} \zeta_\tau \times \mathbb{I}\{t=\tau\} + \sum_{\tau=-3}^{3} \alpha_\tau \times reallocated \times \mathbb{I}\{t=\tau\} + \xi_t + \theta_{ik} + \epsilon_{ikt}, \tag{12}$$

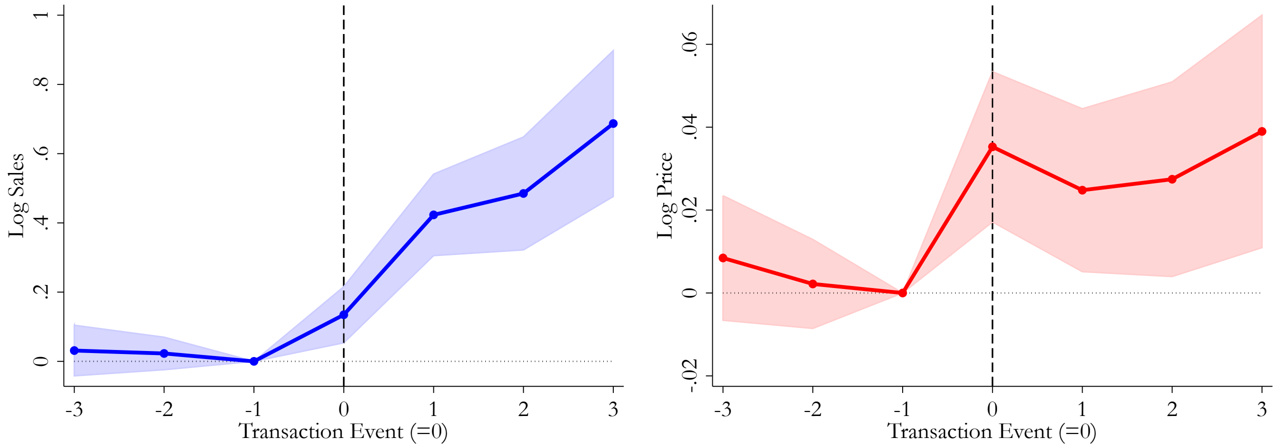
where the unit of analysis is at brand i within product group k in the year t. Above, y_{ikt} is either the total revenue or the average price. We define the average price of a brand-group-year cell by aggregating all transactions within this cell by taking the sales-weighted geometric average of transaction-level prices. *reallocated* is an indicator variable for whether the brand belongs to the reallocated group. ξ_t is a year fixed effect; θ_{ik} is a brand-group fixed effect. τ is an event indicator around the time of a reallocation event. ζ_τ measures the time-level averages for all brands in the sample (treated and untreated).

The coefficients $\{\alpha_\tau\}_{\tau=-3}^{3}$ are the coefficients of interest. They estimate the average difference of prices and sales (y_{ikt}) for reallocated brands compared to similar brands, as selected by the coarsened exact matching algorithm, around the time of the event. Figure 4 plots the estimated $\{\alpha_\tau\}_{\tau=-3}^{3}$ for the regressions for sales (panel A) and the regression for prices (panel B).

We discuss two main takeaways from Figure 4. First, the reallocated brands experience an increase in sales relative to their unreallocated counterparts. After 3 periods, sales are up around 60% from the pre-transaction date compared to similar brands in the same product group, averaging 43% over those post-event periods. This indicates an expansion of revenue for the reallocated brands relative to a comparable set. Interpreting these results through the lens of our model, we argue that the average reallocation events are towards more efficient producers (the frontiers within their groups).

Second, there is an increase in prices of the reallocated brands compared to their counterparts. On

Figure 4: Brand Transactions and Matched Pairs



(a) Sales × Transaction

(b) Prices × Transaction

Notes: Coarsened exact match coefficients. Match is conditional on brands opening with at least $10,000 in sales. Match is made on pre-trend sales, exact product group, and exact year. 95% confidence interval standard errors clustered at the brand-group level.

its own, an increase in prices could suggest a less efficient acquiring firm or higher markups. Given the increase in sales, buying firms appear to exhibit advantages relative to the selling firm. Interpreting these results through the lens of our model, we argue that markup differences between the selling and buying firms play a role. Although the average reallocation is towards more efficient firms, it is also towards the firms with higher markups. The markup effect outweighs the efficiency gain for the average reallocation event, as prices are higher after the event.

Even if the acquiring firm were significantly more productive on the supply side, these gains are not passed on to the consumer likely due to an increase in markup. The distinct outcomes for sales and prices suggest two mechanisms. First, brands are reallocated to firms who expand sales relative to the existing incumbents, providing a scale advantage, a^D. Second, there is tension in this process, since acquiring firms both expand sales and raise prices, suggestive of a rising markup.[15]

Event Study: Averages and Heterogeneity. While our event study outcomes report the average effect of reallocation, this average effect masks significant heterogeneity depending on buyer and seller characteristics and the nature of individual transactions.[16] Table 4 reports this heterogeneity, focusing on the average effect of a reallocation comparing the change in prices and sales three years after the event compared to one year before the event and other brands in the same product group at the same time.

Relative to the average sales change in a brand in that group in the event year, 69% of transacted brands experience an increase in sales after they are reallocated, whereas 54% of transactions experience an increase in prices.

[15]The same qualitative result can also be found at the UPC-level for brands that maintain the same exact product codes, which we discuss in Appendix C.4.

[16]Much of this discussion of heterogeneity we leave for Appendix C.4.

Table 4: Event Breakdown: Averages and Heterogeneity

	Sales	Price		Share
Averages			*Heterogeneity*	
Baseline	0.43	0.032	+ Sales / - Price	0.29
Weighted	0.13	0.010	+ Sales / + Price	0.40
Select on Big Buyers	1.88	0.049	- Sales / + Price	0.14
			- Sales / - Price	0.17

Notes: Baseline selects cutoff at $10,000 in initial sales. Weighted version and big buyers takes all variables. Big Buyer is defined as the top 10% of buyer market share, which is approximately 0.08. Source: USPTO and RMS Nielsen.

The tension between productive and strategic reallocation can be seen in the heterogeneity of outcomes, as some transactions exhibit what looks like purely a productivity gain (only sales go up, and prices stay flat or decline). In contrast, others exhibit a purely strategic effect (prices go up, with a negative effect on sales). 29% of transactions exhibit a price change below the average and a sales change above the comparison group (primarily productive effect). 14% of transactions exhibit a sales change below the average and a price change above the comparison group (primarily strategic effect). 40% of transactions show evidence of both strategic and efficient forces. We revisit these details in Section 4, where we quantify our model.

Retail Expansion and Customer Acquisition. We also aim to understand which margins of sales expand after a brand reallocation event. We find that acquiring firms are more likely to expand the brand into different retail establishments, consistent with the acquiring firms expanding customer base. We also find minimal evidence of immediate exit at the customer level, indicating the persistence of customer capital even after reallocation. Appendix C.4 provides more detail on both of these margins of adjustment.

Local Market Effects. To further understand the interaction between market power and brand reallocation, we look at local markets or specific retailer markets. In the model, we find that firms with a larger presence in product markets have higher markups. The following empirical exercise answers this question: how does the change in market share interact with a firm's pricing decision after a reallocation event?

We look at the change in prices at the brand-parent store level in order to understand how market share will predict changes in prices. We evaluate the how the price at the brand-level changes when a brand is reallocated across firms, given the change in firm shares observed during this reallocation:

$$\Delta y_{ijt} = \alpha + \beta \Delta firm\_share_{j,j',t-1} + \lambda_t + \xi_j + \epsilon_{ijt}.$$

We focus on the price change in four columns in Table 5. In Column 1 and Column 2, we look at the change in price with respect to firm market share, without sales-weights and with sales-weights, respectively. In Column 3 and Column 4, we look at the change in sales against firm market share.

We find that prices tend to move positively with firm market share, consistent with larger firms exerting more upward pressure on prices, as has been noted in the literature (e.g., Amiti et al., 2019). For a firm with a 1 percentage point higher market share, prices for a given brand are 1.19% or 1.53% higher, conditional

Table 5: Local Firm Market Share and Change in Prices and Sales

	(1)	(2)	(3)	(4)
	Δ Log Price	Δ Log Price	Δ Log Sales	Δ Log Sales
Δ Firm Market Share	1.19**	1.53**	-2.07	-1.21
	(0.44)	(0.51)	(4.50)	(3.64)
Sales-Weights	No	Yes	No	Yes
Firm, Retailer, Year Fixed Effects	Yes	Yes	Yes	Yes
N	10700	10683	10700	10683
R^2	0.423	0.435	0.224	0.269

Standard errors clustered at the firm level in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

on the same firm and year. In terms of sales, we don't find significant effects, indicating a potential tension since large firms have less incentive to expand sales but more capability to do so. Larger firms don't necessarily expand sales in markets where they already have prominence.

The goal of these regressions is to express the mechanism of the variable markup depending on a firm's market power. Acquisitions expand sales, but less so within markets where firms already have significant share. On the other hand, higher prices in markets with significant share indicate the role of the strategic effect for firms. We now turn to the interaction between the data and the model in the next section.

4 Estimation

In this section, we unite the model and the data with an estimation procedure that links fundamental parameters to observed objects in the data. We leverage variation at the firm and brand transaction level to match model-generated moments. The estimation delivers a set of moments that were not directly targeted. After estimating the model, we turn to these untargeted moments and compare the model-generated moments to the data.

4.1 Targeted Moments

We calibrate our model to the moments in steady state, but evaluate the transitional dynamics in our policy counterfactuals. We calibrate 10 parameters. We start by discussing the external parameters, three of which we take from the literature. We then discuss seven parameters, which are calibrated jointly with simulated method of moments (SMM).

Externally Calibrated Parameters. We use values from the literature to calibrate a subset of our parameters. The model is calibrated to an annual frequency and we set the discount rate $\rho = 0.03$. We calibrate the within-group substitution elasticity $\sigma = 3.9$. This value is the median across-firm substitution elasticity from Hottman et al. (2016), estimated from a similar demand system and RMS Nielsen Scanner data. For the across-group substitution elasticity, we set $\theta = 1.1$. This is close to the literature benchmark of Cobb-Douglas but allows for some cross-product group competition. We test robustness of this parameter in Section 7 and find the main results hold. These elasticities imply the minimum profit margin of firms is around 0.25, and the maximum is 1.0. The substitution elasticity σ has important implications for our counterfactuals, as the degree to which consumers substitute across brands affects the incentives of firms. We report the quantitative analysis given different values of substitution elasticity.

Internally Calibrated Parameters. In addition to our externally calibrated parameters, parameters related to the following processes are crucial for our quantitative analysis: the *creative destruction* γ at which a vintage firm is replaced by a new frontier firm,and the frontier firm's step size advantage a. We also focus on the *cost of brand reallocation intensity*, parameterized to be $R(\lambda) = r_0 \lambda^{1+\frac{1}{r_1}}$, where we refer to r_1 as the reallocation elasticity and r_0 as the cost shifter of reallocation. We are interested in the *volatility of brand diffusion*, ν. This explains the excess variation in brand capital at the firm level not due to reallocation or entry. While each of these parameters is jointly estimated, we provide a heuristic outline of identification in the following paragraphs.

Brand reallocation events provide a opportunity to separate the firm productivity difference a from the brand gap b. The reduced-form estimates on sales and prices from the event studies include changes in appeal, productivity, and markup from the lens of our model. We utilize the model-implied equations to back out these changes. From the model, when a brand changes ownership, the sales of the focal brand change by:

$$\log \text{Sales}_{it+1} - \log \text{Sales}_{it} = a_{k(i)t} + (\sigma - 1) \log \frac{\mu_{J(i,t+1)}}{\mu_{J(i,t)}},$$

which comes from the change in productivity and change in markups. With the formula for markups at the firm level, we use the pre-reallocation market shares at the selling and buying firms to impute the change in markups. The residual change in revenues provides delivers the value a. This distribution could directly inform the evolution of a if brand reallocation events always went to the most productive firm. However, as in our model, these events can go in the opposite direction of the productive firm. We thus jointly calibrate (a, b_0, ν) to match the average effect and the share of events that lead to positive sales change. This leaves us with a b_0 and ν to match the share of positive events, jointly with other moments and parameters discussed below. We evaluate the robustness of our model's positive and normative implications in Section 7.

For the *reallocation process*, we assign three parameters to the model. First, we calibrate the size of a brand reallocation event, Δ, to match its data counterpart of 0.111 in log revenues.[17] We then calibrate the constant in the reallocation cost function, r_0, to match the shares reallocated per year, 0.013. Intuitively, a higher r_0 means it is more costly to increase the reallocation intensity. Thus, the average reallocation event size is informative about r_0. Secondly, we calibrate the elasticity of reallocation, r_1, to match the standard deviation of market share reallocated in a year, 0.23.

In summary, we have six parameters to estimate jointly, $\{a, \nu, b_0, r_0, r_1, \gamma\}$. These parameters match the share of positive events, average sales change of reallocation events, the reallocation-size correlation, and the residual variance of firm growth and the creative destruction to match overall economic growth.

[17] This measures the firm market share reallocated as a share of the total of the two firms transferring brand capital.

Table 6 delivers 10 parameters that provide the foundation of the model. We will further analyze these parameters by reporting their ability to match out-of-sample moments in the next section. We will then report policy counterfactuals and the robustness of these outcomes to variation in the parameters in Section 7.

Table 6: Estimation Moments and Parameters

Parameter		Value	Moment	Data	Model
Independently Calibrated					
Discount Rate	ρ	0.03	Annual Risk-free Rate		
Substitution Elasticity					
Within	σ	3.90	Hottman et al. (2016)		
Across	θ	1.10	Hottman et al. (2016)		
Jointly Estimated					
Creative Destruction					
Step	a	1.54	Sales Event Study	0.43	0.43
Rate	γ	0.03	Growth Rate	0.02	0.02
Initial Brand Gap	b_0	-1.34	Shr. Pos Events	0.69	0.69
Reallocation					
Reallocation step size	Δ	0.111	Average Event Size	0.111	0.111
Cost - Level	r_0	3896	Average Share Reallocated	0.013	0.013
Cost - Elasticity	r_1	0.18	Std. Share Reallocated	0.23	0.23
Brand Diffusion Variance	ν	0.74	Residual var. in growth	0.01	0.01

Notes: Parameters estimated separately (top panel) and jointly (bottom panel). Source: RMS Nielsen, USPTO and author calculations.

4.2 Untargeted Moments

There are several empirical regularities that are not directly targeted by our estimation. This section uses some of these regularities as out-of-sample tests for the relevance of our mechanism. We present these untargeted moments in Table 7.

Table 7 presents 5 untargeted moments in terms of data and model-generated values. We discuss each of the main untargeted moments below.

Table 7: Untargeted Moments, Summary

Outcome of Interest	Model	Data
Leader/Follower Market Share Difference	0.25	0.30
Leader Persistence	0.98	0.92
Event Study Log Prices	0.066	0.032
M&A Premium (Assuming $\phi = \frac{1}{2}$)	0.44	0.47 (David, 2020)
Residual Event Variance, Sales	0.39	0.83

Our model incorporates the turnover of market share amongst large firms. In our estimation, we deliberately leave the persistence of product group leadership as an untargeted moment, in order to verify the empirical relevance of our mechanism. The persistence of firms' leadership is determined by both the creative destruction process and the brand diffusion process in our model. We calibrate the diffusion to match the residual growth of firms and the reallocation to match the reallocation flows. Our calibration did not use the persistence of leadership as a direct target. In the data, the persistence of group leadership is 92%, meaning the product-group leader in the past year has a 92% probability of staying the leader in the current year. In our estimated model, the predicted persistence is 98%. Although not perfectly, our model does provide a good fit for the observed persistence in group leaderships.

The second untargeted moment we consider is the average effect on prices. In our estimation, the two parameters that govern markups, (θ, σ), are taken from the literature, while the parameter that governs the productivity gain, a, is used to match the average sales effect. In our empirical analysis, we find an average impact on prices of 0.032. Our estimated model predicts an average price change of 0.078. While the model overstates the markup effect, this is based on the assumption that the variation amongst firms in their productivity comes from the appeal, and thus may underestimate the cost advantage of the acquiring firm.

We follow the firm dynamics literature to parameterize the diffusion process of brands as Brownian motion, which implicitly imposes that the stationary distribution of brand gaps is normal when we ignore the endogenous reallocation process. This parametric assumption could be tested against the data, when we consider the distribution of sales effects across different events. We thus use the dispersion of sales impacts as an untargeted moment. In the data, the dispersion in the sales impact is 0.83. In our model, this dispersion is 0.83. While it is not exactly matched it qualitatively coheres with the dynamics at the firm level matched within the model.

The last untargeted moment reports the M&A premium from the literature (David, 2020), and compares this to the premium from the model, which would be the gains from trade. The two numbers are a close fit, indicating coherence with the literature on M&A, even with two types of capital.

5 Is Brand Reallocation Welfare Enhancing?

The welfare implications of brand reallocation are ambiguous in the theoretical model. Our estimated model is able to quantify this welfare incidence. The core of this quantification is a decomposition of brand reallocation events. According to our model, events where brands expand can lead to either an increase in markup or a decrease in markups.

We proceed by decomposing the events in Table 8. In this table, we evaluate four types of events: when sales increase, it can correspond to productivity and markups increasing, or productivity increasing and markups decreasing. When sales decrease, this can be due to productivity increasing and markups increasing enough to offset, or due to a pure reallocation to the "wrong" firm, $a \downarrow$ and $\mu \uparrow$.

Table 8: Decomposition of Reallocation Events

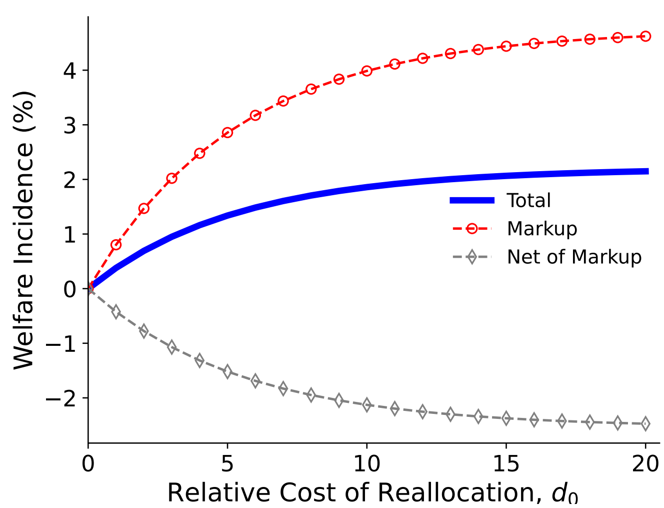
Type of Events		Model	Model Total	Data
Data Type	*Model Type*			
Sales Increases	$a \uparrow$ and $\mu \downarrow$	0.13	0.69	0.69
	$a \uparrow$ and $\mu \uparrow$	0.56		
Sales Decreases	$a \uparrow$ and $\mu \uparrow$	0.11	0.31	0.31
	$a \downarrow$ and $\mu \uparrow$	0.20		

There are some key quantitative results from the table. First, our model exactly matches the share of transactions that have positive and negative sales in the data. Second, we find two different types of transactions when sales decrease. In the first case, productivity a increases, but this is offset by a larger markup μ increase. This is a standard discussion amongst antitrust economists and occurs in 11% of the transactions. The second case illustrates the importance of brand capital, as brands are reallocated to the *less* efficient firm. This occurs in 20% of the transactions. This mismatched case is sizable and indicates how many markets have brand persistence leading to longer-run inefficiencies. Overall, 42% of transactions are inefficient, either due to markups or mismatch.

We next turn to the overall welfare incidence of brand reallocation in response to changing the cost of reallocation. To answer this question, we gradually increase the cost of reallocation d_0 to a large number. Figure5 reports this comparative statics. In this figure, we report the relative welfare, in consumption equivalence, to the baseline calibration. As the cost of reallocation increases, the total welfare increases monotonically, indicating brand reallocation leads to overall distortions in the economy. In the limit, the economy gains in welfare of 1.4%, which is our headline number of welfare incidence.

We further decompose this welfare incidence into two components: the welfare changes due to markup distortions and the welfare changes due to other forces. While the brand reallocation does improve the sorting between brand capital and firm productivity (2.9%), this welfare gain is not materialized into utility. The welfare loss due to markup distortions (-4.3%) eventually turns the welfare gain into a welfare loss.

Figure 5: Welfare Incidence of Brand Reallocation



Notes: This figure plots the welfare difference between the baseline case and the relative cost of reallocation d_0 as percentage points. We vary d_0 (x-axis) and look at the welfare incidence (y-axis) in the baseline (blue line) and netting out the effect of markup (gray line). *Source*: author calculations.

6 Policy Implications

By introducing brand capital and brand reallocation into a standard oligopolistic model of firm competition, we provide a new angle for analysis of standard policy counterfactuals. Here, we perform analysis of different policy counterfactuals. As a benchmark, we start by setting up the planner's problem in our economy. This will serve as the main reference point for our policy analysis. We show that the gap between the decentralized equilibrium and the planner's allocation reflects the static markup distortion and the dynamic mismatch between firms' brand capital and core productivity.

6.1 The Planner's Solution

The planner makes both static decisions and dynamic decisions to maximize the representative household's discounted utility. Statically, she chooses how much firms produce given their productivity and their brand capital. Dynamically, she chooses how intensely to promote reallocation events by choosing the arrival rate for reallocation λ and the direction of reallocation. The planner aims to sort brand capital to the frontier firm and solve the static inefficiency from markups.

In the static allocation of production, the social planner chooses production (and thus, consumption) given the distribution of brand gaps across product groups. The optimal static labor allocation is standard. Compared to the equilibrium, there are no markup distortions in the planner's allocation. Two implications follow. The planner sets the production labor to 1, and the aggregation consumption equals the aggregate productivity $\mathbf{C} = \mathbf{Z}$, where $\mathbf{Z}$ is defined in Lemma 1.

We now turn to the planner's dynamic decision, which is where the role of brand capital leads to novel insights. The planner chooses the arrival rate of reallocation events, $\lambda(b)$, and the direction of reallocation, $\iota(b)$, to maximize the discounted utility of the representative household as follows,

$$\mathbf{W}^* = \max_{\lambda(b),\iota(b)\in\{-1,1\}} \int_0^\infty \int_{-\infty}^\infty e^{-\rho t} \left[\log \mathbf{Z}_t \mathbf{L}^P - \mathbf{L}^P - \mathbf{R}_t\right] dt, \tag{13}$$

s.t.

$$\dot{g}(b) = -\lambda(b) g(b) - \frac{\chi^2}{2} g''(b) + \lambda^+(b-\Delta) g(b-\Delta) + \lambda^-(b+\Delta) g(b+\Delta). \tag{14}$$

In Equation (13), the flow payoff of the household depends on the aggregate productivity $\mathbf{Z}_t$, the measure of production labor $\mathbf{L}^P$ (which is 1 according to the static optimal allocation), and the amount of reallocation labor, $\mathbf{R}_t$. The aggregate productivity $\mathbf{Z}_t$ depends on the distribution of brand gaps, $g_t(z)$. This dependence is given by the aggregation result in Lemma 1. The planner faces the law of motion of brand gaps, which is given by Equation (14).

This dynamic optimization problem can be characterized by a Hamilton-Jacobi-Bellman (HJB) equation resembling the joint surplus equation of duopolists in the equilibrium, as in Equation (9). The only difference between the planner solution and the equilibrium surplus is in the static payoff:

$$(\rho+\gamma)\Omega^*(b) = \omega^*(b) + \max_{\lambda} \lambda \max\left\{\Omega^*(b+\Delta) - \Omega^*(b), \Omega^*(b-\Delta) - \Omega^*(b)\right\} - R(\lambda) + \gamma\Omega^*(b_0) + \frac{\nu^2}{2}\Omega^{*\prime\prime}(b), \tag{15}$$

where

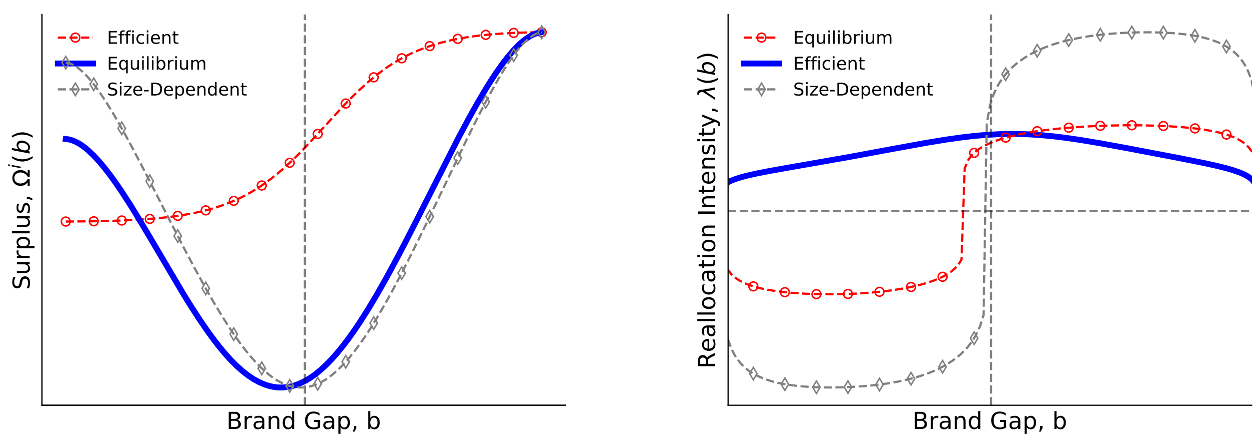
$$\omega^*(b) = \frac{1}{\theta-1}\left(\frac{Z(b)}{\mathbf{Z}^*}\right)^{\theta-1}.$$

In Equation (15), the flow utility of having a product group with brand gap b is given by $\omega^*(b)$. There are two disparities between the planner's and the firms' Bellman equation. First, the planner internalizes the

the full consumer surplus, which is larger than the profit margin of firms ($\frac{1}{\theta-1} > \frac{1}{\theta}$); Second, the planner does not face the distortions due to markups.

The planner has very different incentives from the firms, which can be seen in Figure 6. This figure plots a normalized surplus of the firms in equilibrium, a policymaker that institutes a size-dependent policy, and the planner optimal. We demonstrate the efficient and equilibrium allocations and reallocation decisions. Figure 6a focuses on the surplus of the planner, who targets the efficient allocation (blue) and the firms which maximize their own surplus (red). Figure 6b shows the reallocation decisions, conditional on a reallocation opportunity, of the planner (blue) and the firms (red) in equilibrium.

Figure 6: Comparison between Equilibrium and Planner Allocation



(a) Joint Surplus

(b) Reallocation Policy Function

Notes: The left panel plots the joint surplus according to the planner and equilibrium allocations, according to the baseline calibration. The values are normalized by subtracting the level at $b = -20$; The right panel plots the corresponding reallocation policy function.

We stress a few takeaways from this figure. First, Figure 6a shows the difference between the planner's surplus and the equilibrium surplus. When the brand gap is low, the firms prefer consolidation as it enables a larger profit margin. This, however, keeps brand capital in the inefficient vintage firm. The planner wants the more productive firm to hold the brand capital, so the planner gains from increasing b. This comes from sorting the brand capital to the more productive firm.

Figure 6b illustrates the reallocation decisions in equilibrium and for the planner. First, the planner is always incentivized to reallocate to the frontier firm (blue line). However, when brand capital is sorted to the vintage firm ($b < -a$), the firms in equilibrium sort brand capital to the inefficient leader to again consolidate the profit margin. Once brand capital crosses the productivity threshold ($b > -a$), the planner and the firms in equilibrium have similar incentives to consolidate brand capital to the more productive firm conditional on reducing the markup dispersion. Overall, these figures illustrate the distinct strategies of the firms in equilibrium and the planner in particular when brand capital is sorted to the vintage, less productive, firm.

This divergence between the planner and the firms in equilibrium also has implications for policies. Whether a policy is optimal depends on how it accords with the planner's solution. This hinges on the planner's knowledge. This will be a key point in revisiting standard size-dependent policies with the role of brand capital.

6.2 Size-Dependent Policies in the Presence of Brand Capital

Our study will start with optimal policies through the lens of a planner with full information. The fundamental static and dynamic distortions in our model arise from the presence of variable markups. A natural question is whether a policy remedy that targets markups on their own is enough to bring the economy to an efficient allocation. Our answer to this question is no. A policy that ignores the sorting between brand capital and productivity can lead to new distortions. In our baseline calibration, these distortions are large enough to offset the welfare gains from resolving markup distortions.

We consider three alternative polices. First, we focus on an unconstrained planner that can solve the pricing inefficiency and the brand reallocation inefficiency. This planner is aware of the joint distribution of brand capital and productivity. Second, we focus on a planner that can solve the pricing problem alone, in line with current policies in the literature (Edmond et al., 2023). Third, we focus on a planner who can only determine the allocation of brand capital across firms but can not solve the static pricing problem. These three scenarios enable us to understand the nature of the misallocation. We convert the size and reallocation-based policies into subsidies to map these scenarios onto policies.

Once brand capital and productivity are considered to jointly determine market share, classic policies under monopoly have different implications. We compare the planner's solution to two alternative policies: *size-dependent subsidy* and *reallocation taxes and subsidies*. These two policy instruments are discussed substantially in the literature as remedies for the distortions from firm markups. Due to the disconnect between market leadership and productivity, our quantitative analysis shows that a blunt size-dependent subsidy could backfire.

Size-dependent policy. With a size-dependent policy, we consider a subsidy that implements the static optimal allocation when the policy maker only has data regarding the market shares which are a function of effective productivity $z = a + b$, but not (a, b) separately. Following a similar logic as in Edmond et al. (2023), we derive the optimal subsidy schedule such that the firms produce at the static social optimal. In the appendix, we show that the joint profit of the duopolists can be written in closed form in our economy:

$$\omega^z(b) = \underbrace{\frac{\sigma}{\sigma - 1} \log \frac{(1 + e^{a+b})^2}{e^{a+b}}}_{\text{Within Group}} \times \underbrace{\frac{Z(b)^{\theta-1}}{(\mathbf{Z}^s)^{\theta-1}}}_{\text{Across Group}} \,.$$

We showcase the joint profit of the firms under the optimal size-dependent policy in Figure 6. Although this size-dependent policy corrects the static distortions due to markups, it exacerbates the mismatch between brand capital and productivity. More specifically, under the size-dependent policy, the joint profit function becomes more convex in the wrong direction. Consequentially, firms put more effort into moving brand capitals into the wrong firm.

Reallocation-only policy. The second policy instrument we consider is a reallocation tax/subsidy that maximizes the household's welfare, given the pricing equilibrium outcomes. So under this policy seniario, the policy maker take as given the productivity $Z(b)$ and the markup $M(b)$. She can only rely on choosing the reallocation intensity and its direction to maximize the representative household's utility. One could interpret this as an anti-trust policy where the government cannot directly control prices of firms. For this constrained planner, the static payoff from having an additional product group with brand gap b resembles the one in equation (15). It differs in the markup terms.

$$\omega^r(b) = \frac{1}{\theta - 1}\left(\frac{M(b)}{\mathbf{M}^r}\right)^{1-\theta}\left(\frac{Z(b)}{\mathbf{Z}^r}\right)^{\theta-1}.$$

Decomposition of Welfare Gains. Given the utility function, we can write the steady-state welfare as,

$$\rho \log \frac{\mathbf{W}}{\mathbf{W}^e} = \mathbf{c}_M + \mathbf{c}_Z, \tag{16}$$

where $\mathbf{c}_M = -\log \mathbf{M}^e - \left(1 - \frac{1}{\mathbf{M}^e}\right)$ and $\mathbf{c}_Z = \log \frac{\mathbf{Z}}{\mathbf{Z}^e} - (\mathbf{R} - \mathbf{R}^e)$. Although the full welfare incidences of alternative policies include the welfare gains from the transitional path, the steady-state values offer a tractable way to understand these incidences. Using the formula above, we can decompose the welfare differences between the equilibrium and the policy-induced allocation into the consumption equivalence due to resolving the markup distortions given the distribution of brand gaps and the one due to more efficient allocation between brand capital and productivity.

Results. Table 9 reports the productivity losses in three environments: The first environment is the average case in our sample Hottman et al. (2016), which takes the average elasticity of substitution from their paper. We add a "high-markup" world ($\sigma = 2$) and a "low-markup" environment ($\sigma = 8$), and compare overall welfare under the efficient scenario to the size-dependent policy and the reallocation policy. Consumption equivalent welfare is made up of consumption, labor in production and reallocation labor.

Table 9 presents some core quantitative messages of our paper. We start by focusing on our baseline case. First, we note that the equilibrium diverges significantly (52.3% in consumption equivalence) from the efficient outcome. This gain comes almost entirely from resolving markups. Second, if we implement a size-dependent policy that ignores the brand component of firm size, we reach the same gains from correcting markups. This is not surprising, as both allocations implement the efficient allocation given the stationary distribution of brand gaps. However, this size-only policy brings significant losses in the steady-state welfare due to the mismatch of brand capital. The welfare losses from subsidizing the large firm leads to misallocation of labor with a 38.3% reduction in welfare. This is due to incentivizing firms to spend resources to reallocate brand capital to inefficient firms. These reallocations both cost labor while and decrease aggregate labor productivity through mismatch. We take the loss c_Z and the gain c_M, to show that correcting mismatch accounts for 68% of the restoration of the efficiency in the static allocation.

Table 9: Policy Comparison Table

	Efficient	**Size Dependent**	**Reallocation Only**
Baseline $\sigma = 3.9$			
$Chg.Welfare(full)$,	52.31	13.42	2.30
$Chg.Welfare(s.s.)$,	57.84	17.73	5.00
c_M,	55.97	55.97	7.28
c_Z,	1.82	-38.28	-2.28
High Markup $\sigma = 2.0$			
$Chg.Welfare(full)$,	88.15	21.09	1.22
$Chg.Welfare(s.s.)$,	95.46	27.39	4.72
c_M,	91.86	91.86	5.74
c_Z,	3.59	-64.48	-1.02
Low Markup $\sigma = 20.0$			
$Chg.Welfare(full)$,	18.31	-3.00	0.04
$Chg.Welfare(s.s.)$,	22.51	-6.14	0.61
c_M,	22.39	22.39	1.05
c_Z,	0.11	-28.54	-0.43

Overall, a size-dependent policy has around one-third smaller welfare gains than the efficient benchmark. The reallocation policy increases welfare but is very modest compared to the efficient allocation and size-dependent subsidies.

The elasticity of substitution has important implications for policy counterfactuals. We show that changing the elasticity of substitution does not change the qualitative message of the first panel but changes some quantitative results. In both the low-markup and high-markup environments, we find a large divergence in welfare from the optimal and the equilibrium. In the case of large σ, the size-dependent subsidy backfires. Markets with high σ are ex ante competitive: there is little product differentiation when competition is high. However, these markets can generate large gains to consolidation as firms can effectively reduce the elasticity of substitution perceived by the consumer. Thus, the higher σ is, the more policies can backfire in the presence of brand capital.

Introducing brand capital provides some intuitive adjustments to a standard model. Firms want to accumulate brand capital in a single firm to enable markups, which leads to a dynamic mismatch. Indeed, this dynamic mismatch can be very persistent but may be path-dependent. A firm that gets positive productivity shocks may expand its brand capital and start to acquire additional brands. However, this may work in reverse. If a large firm gets negative productivity shocks, it may maintain its high brand capital through acquisition. More productive small firms may sell off their brands because the large firm shares the surplus extracted from the customer.

This core mechanism changes the nature of many policies from the standard model. The goal of the social planner's problems is to think through feasible policies through the lens of market structure. We turn to policies to further elucidate this mechanism and its distinction from the current state-of-the-art literature. We now turn to robustness and extensions to show the qualitative message of this section holds.

7 Robustness and Extensions

This section first develops insights on the robustness of our results to changes in the parameter set. For example, we study parameters relevant for our benchmark calibration: the initial brand capital of the entrant, the volatility of brand capital, and the elasticity of substitution within and across groups. These robustness show the main qualitative results hold for a broad set of our parameter set. We then build out extensions that illustrate the robustness of our results to various core changes in the model, such as endogenous advertisement and creative destruction.

The robustness focuses on the comparison between the efficient planner and size-dependent policy. The results carry two main messages. First, the gap does not reverse for various parameters. Second, the changes in parameters provide further insights on the identification of welfare.

7.1 Robustness

The main quantification relied on specific calibrated estimates for a set of four key parameters, two which are estimated from the model and two of which are taken from the literature. Given our focus on CPG markets, we may expect other markets to have different features. This section demonstrates that the qualitative messages of our paper hold in a variety of cases. We focus on this by varying the fundamental parameters in graphical form.

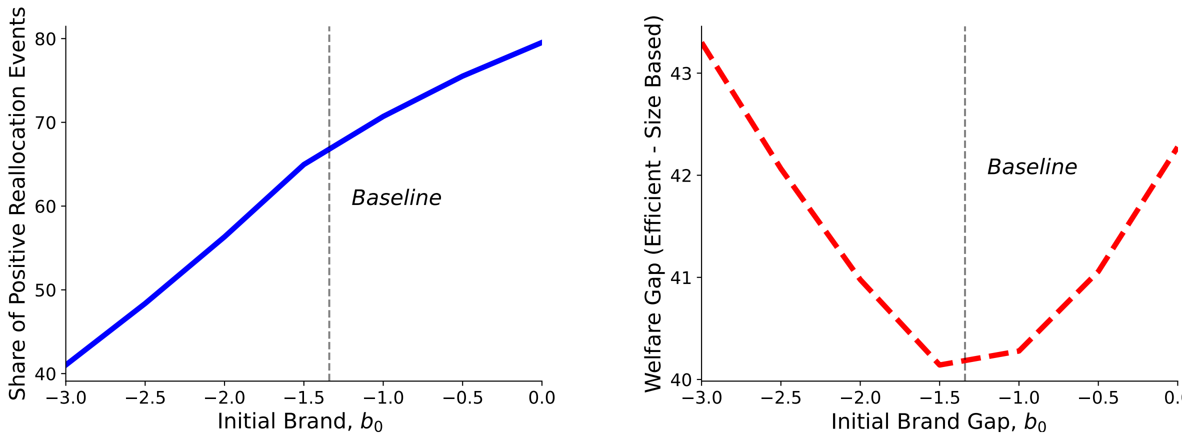
Initial Brand Capital, b_0. We identify the initial brand gap primarily from the share of positive sales events interacted with the identified productivity advantage. However, initial brand gaps may vary significantly across industry and time period. In the first extension, we vary this initial brand gap. This exercise highlights how the share of positive events helps us discipline the parameter and how different values of the initial brand gap matter for our quantitative results.

In the panel (a) of Figure 7, we plot the model-predicted share of positive events along different values of the initial brand gap. We observe that, as the initial brand gap tightens ($b_0 \downarrow$), the share of positive events predicted by the model increases. When the new frontier firm is further behind in brand capital, it becomes more likely for a representative group to experience strategic reallocation events. As a result, the share of events where the brand loses its market share increases.

An initial brand gap that is more negative implies that the share of strategic reallocation increases and the brand reallocation starts to creates negative welfare impacts, and vice versa. Consequentially, the misallocation effect that origins in the size-only policy increases and the welfare gap between the optimal policy and the size-dependent policy widens. In panel (b), we show that for a variety of initial brand gaps, there is a persistent gap between size-dependent and efficient allocation which is increasing as b_0 decreases.

Substitution Elasticity, (θ, σ). The substitution elasticities are taken directly from the literature. We now plot different levels of substitution elasticity and their impact on welfare impacts. In this section, we focus on both the θ and σ elasticities and how changes in these elasticities change the welfare gap. Figure 8

Figure 7: Robustness — Initial Brand Gap



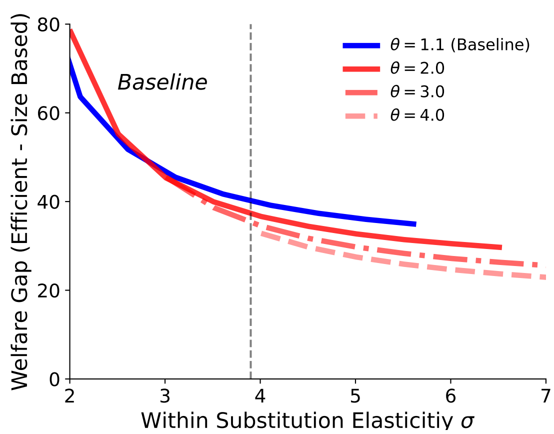
(a) Share of Positive Events

(b) Welfare Incidence

Notes: This figure plots the share of positive reallocation events (panel A) and the welfare gap (panel B) against the initial brand capital, b_0. The welfare gap is defined as the difference between the efficient outcome and a standard size-dependent policy. *Source*: author calculations.

plots the welfare gap between the efficient and size-dependent policies against the substitution elasticity σ for different values of θ, when $\sigma < \theta$.

Figure 8: Robustness — Substitution Elasticity

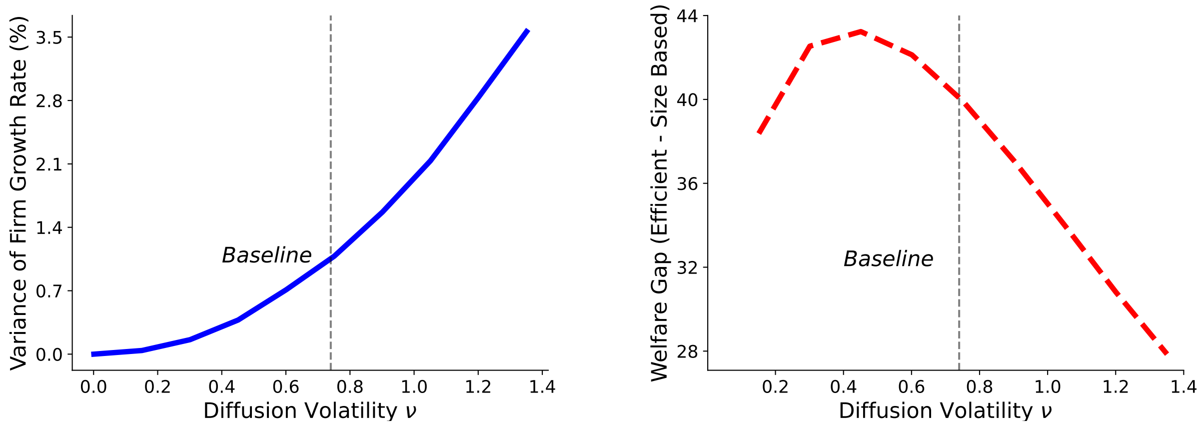


Notes: This figure plots the gap between the efficient policy and size-dependent policies as percentage points. We vary σ (x-axis) and look at the welfare gap (y-axis) for different values of across-group elasticities (θ). *Source*: author calculations.

As θ increases, the share of strategic events decreases. When products from different groups become more substitutable to each other, firms face more outside competition. The markup gains becomes less important because the inefficient allocation of brand capital makes the product group loses market shares. As a consequence, firms are more likely to reallocate towards the direction that enhances productivity. This means the welfare gain from implementing the efficient policy shrinks, and the size-dependent policy has a smaller gap when compared to the efficient allocation. In general, we find the welfare gap from our main result holds qualitatively for a variety of σ and θ.

Brand Diffusion, ν. The brand diffusion variance is calibrated to match the residual variance in firm growth. We now plot different values of such diffusion variance. A lower diffusion variance implies that the product groups are more likely to be stuck in the inefficient states. To see this, we consider a polar case where $\nu \to 0$. This polar case is similar to our analytical results. In this case, all reallocations are either productive or strategic. Coupled with the baseline estimation that $b_0 + a < 0$, this implies that the negative impacts of brand reallocation on welfare are larger.

Figure 9: Robustness — Diffusion Volatility



(a) Firm Growth Variance

(b) Welfare Gap

Notes: This plots the variance of firm growth as a function of the diffusion volatility (panel a) and the welfare gap as a function of the diffusion volatility (panel b).

Figure 9 shows that for a variety of diffusion volatilities the same message holds. However, as exogenous diffusion increases the gap narrows, due to the correcting nature of large shocks in the market.

Frontier Productivity Advantage, a. We identify the productivity advantage from the change in sales in the scenario of positive events. Due to the heterogeneity, one might want to understand the nature of markets with very different leader advantages. In Figure 11 in the next section, we show that our model has very similar results for a range of a in the world with creative destruction. While changes in a have slight effects on the magnitudes, the qualitative points on the welfare gap remain the same.

7.2 Extensions

We focus primarily on two extensions to the baseline model. In the first extension, firms can endogenously invest in brand capital, which may alter some predictions from the model given this important margin. In the second extension, we focus on endogenous entry, where policies that focus on entry will pick up relevance.

Endogenous Advertisement. In this section, we extend our baseline model to consider the role of endogenous advertisement. When firms can endogenously invest in their brand capital, the rate at which brands diffuse differs under alternative policy regimes. This rate further interacts with the endogenous reallocation decisions. Additional distortions show up in an equilibrium with endogenous advertisement as well. First, frontier firms can be discouraged from investing in their brand capital due to the discouragement effect from variable markups, as in Aghion et al. (2005). Second, firms can invest excessively in their customer base due to the zero-sum feature of customer attention.

The extension adds one main component to our baseline model. Firms can choose the rate at which the brand capital evolves. Specifically, the frontier firm can invest in its brand capital with intensity η_1 and the vintage firm can invest in its brand capital with intensity η_2. With this endogenous investment, the net drift of the band gap is $\eta_1 - \eta_2$. When the frontier firm advertises more than the vintage firm, $\eta_1 > \eta_2$, the brand capital gap grows, and vice versa. Advertising is costly. We assume the labor cost is, $D(\eta_1)$ for the frontier and $D(eta_2)$ for the vintage firm. D is increasing and convex, with $D(0) = 0$. The assumption that the advertising cost function is identical implies that the only difference in advertising intensity reflects the difference in the benefits of advertisement. We modify the value function in Equation (7) and Equation (8) to reflect the additional choice of advertising. For the frontier firm, this new value function is:

$$rV(b) = \max_{\eta_1} \Pi(b) + \phi\bar{\Omega}(b) + \gamma(v(b_0) - V(b)) + (\eta_1 - \eta_2)\, V'(b) - D(\eta_1). \tag{17}$$

For the vintage firm, this value function is:

$$rv(b) = \max_{\eta_2}\ \pi(b) + (1-\phi)\bar{\Omega}(b) + \gamma(0 - v(b)) + (\eta_1 - \eta_2)\, v'(b) - D(\eta_2). \tag{18}$$

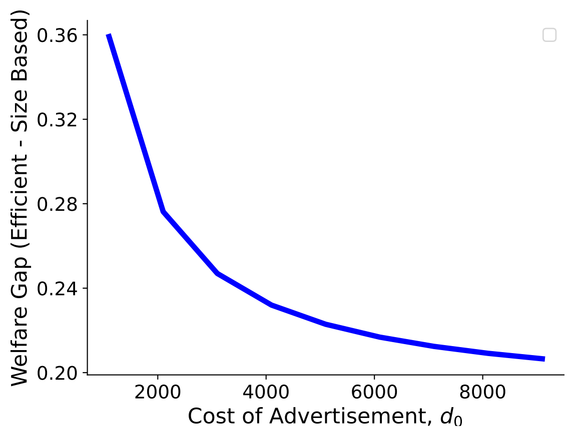
By taking the first order condition, we derive the optimal choice of advertising choices:

$$D'(\eta_1(b)) = V'(b), \quad D'(\eta_2(b)) = v'(b). \tag{19}$$

In their optimal advertising decision, both the frontier and the vintage firm equalize the marginal return to the value of changing the brand gap to the marginal cost of advertising. In the polar case without variable markups ($\theta \to \sigma$), the only difference in the advertisement choice reflects the productivity differential between the two firms. In the polar case without productivity differentials ($a \to 0$), the only difference in the advertisement choice reflects the markup differentials between the two firms. In the cases outside of these extremes, firms' advertising choices reflects both the productivity differentials and the markup differentials. This comparison echos the discussion of reallocation choices: when the sizes of firms are determined jointly by productivity and brand capital, advertising could reinforce or weaken the sorting. We show that the endogenous advertisement exacerbates the gaps of policy incidences when we compare the efficient allocation to the size-dependent policies.

We showcase the implications of endogenous advertisement using our baseline parameters with an additional advertisement cost function $D(x) = d_0 x^2$. In Figure 12, we re-calculate the welfare gaps between the efficient outcomes and a size-dependent policy, for different levels of d_0. In the calculation, we assume the bargaining power of the two firms is both $\frac{1}{2}$. Endogenous advertisement amplifies the distortions

Figure 10: Extension: Cost of Advertisement and Efficiency



Notes: This figure plots the gap between the efficient policy and size-dependent policies as percentage points. We vary d_n (x-axis) and look at the welfare gap (y-axis) for the model with endogenous advertisement. *Source*: author calculations.

introduced by a size-dependent policy. This distortion widens when the advertisement becomes cheaper. These results paint a complex picture of the role of advertising in brand development and market share. On the one hand, advertising can indeed sort consumers to the most productive firm. On the other hand, advertising can serve to solidify the incumbent advantage for *less* productive incumbents than their competitors. The role of advertising depends on history, persistence, and the elasticity of substitution of consumers.

Overall, advertising in our model also exhibits similar features to the role of advertising discussed in the literature (Cavenaile et al., 2022). There is a role of *net* advertising which sorts consumers across firms, and *gross* advertising which uses up resources in the economy to try to capture the attention of consumers. The business stealing effect (the relationship between gross and net) thus plays a role in the social planner's decision, who would generally choose no advertising for firms with lower productivity in each market.

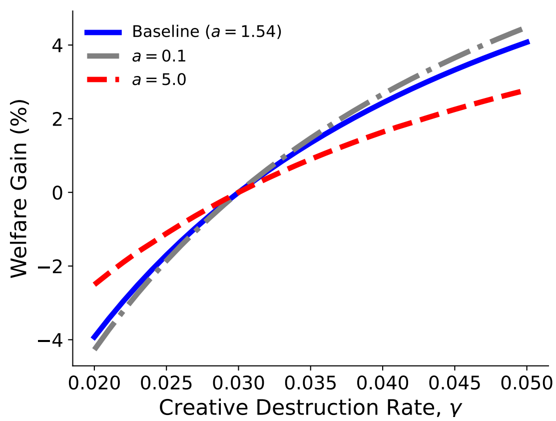
Endogenous Creative Destruction. Our current framework does not endogenize innovation and directs primary attention to the development of brand capital. Although there are many other forces that shape the growth rate of the economy, it is useful to understand the interaction between the endogenous growth mechanisms and brand capital evolutions. In this section, we consider an extension where the creative destruction rate λ is determined by the endogenous innovation choice of new entrants. In doing so, we also discuss the role of entry subsidy policies in our model.

Suppose there is a large measure of potential entrants who can pay a fixed labor cost κ to innovate. They succeed in innovation with a rate $\tilde{\lambda}$, which we directly calibrate from the endogenous growth literature. When an entrant succeeds, a creative destruction event happens, and the product group progresses as in our baseline model. We focus on the case where there is a positive entry, so the entrants are indifferent about whether to innovate. The optimal entry decision thus requires that, for any product group with brand gap b,

$$\kappa = \tilde{\gamma} V(b_0).$$

We fix other parameters as in the baseline calibration and vary the entry κ. In this model, the creative destruction rate is an equilibrium object, $\gamma = \tilde{\gamma} n$, where n is the endogenous measure of entrant firms. Different κ leads to different levels of creative destruction rate. The equilibrium rate γ should lead to the free-entry condition with equality. The following graph plots the welfare implications of changing κ (and correspondingly γ) and changing a.

Figure 11: Extension — Productivity Gap and Creative Destruction



Notes: This figure plots the gap between the efficient policy and size-dependent policies as percentage points. We vary γ (x-axis) and look at the welfare gap (y-axis) for different values of average productivity gap (a). *Source*: author calculations.

The result on creative destruction shows that more market dynamism leads to higher welfare, regardless of a. However, the larger the gap the flatter the relationship between creative destruction and welfare.

8 Conclusion

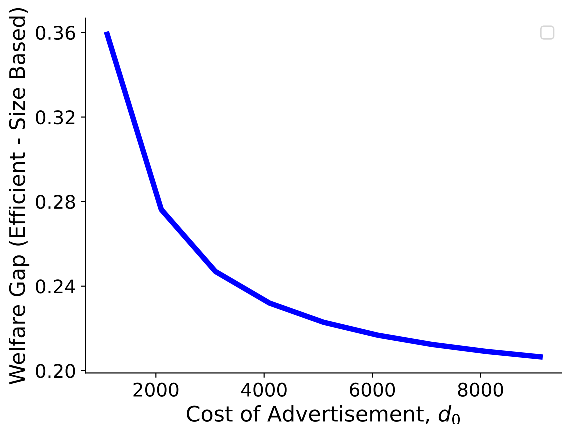
Customer capital and productivity jointly determine the market share of firms. Brand capital, as the tradable component of customer capital, has large capital value and is constantly exchanged across firms. We build a new theoretical model and dataset to study the interaction between brand capital and productivity.

Our paper discusses novel theoretical, empirical, and quantitative results. Theoretically, We find that brand reallocation can generate a dynamic mismatch between productivity and customer capital when variable markups are present. As a result, brand reallocation can have an ambiguous welfare impact and are susceptible to path dependency. Firms with higher brand capital from the past value the marginal brands more. Empirically, we find that brand reallocation is important for firms to maintain their existing market shares. When brands are reallocated, there is evidence of both productive and strategic incentives.

Quantitatively, we find that brand reallocation on net reduces welfare. We also find that the size-dependent policies from the literature can be distortionary once the dynamic mismatch between brand capital and productivity is considered.

Our results highlight the importance of understanding the sources of firm performance beyond the single-dimensional productivity measure. Whether market power comes from productivity or accumulated customer capital leads to different conclusions regarding the efficiency of the market economy. We

Figure 12: Extension: Cost of Advertisement and Efficiency



Notes: This figure plots the gap between the efficient policy and size-dependent policies as percentage points. We vary d_n (x-axis) and look at the welfare gap (y-axis) for the model with endogenous advertisement. *Source*: author calculations.

discuss some possible extensions of our framework here. First, we studied brand reallocation without explicitly quantifying the growth implications. For more general welfare criteria, this should be taken into account. Second, the interaction between the opportunity to reallocate and entry is assumed away, which could also bear quantitative importance. Firms may innovate and enter given the option value of brand reallocation. Lastly, there could be other sources of firm decisions that lead to similar patterns of mismatch, such as political connections. We believe these discussions are fruitful extensions for further research.

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Appendix

The Appendix contains three sections. Appendix A discusses the theoretical proofs and expands on the firm's dynamic problem. Appendix B discusses the data background and merges across datasets. Appendix C discusses the empirical analysis, connections to the literature, and empirical robustness.

A Theoretical Appendix

In this section, we discuss the omitted proofs and planner's solution in turn.

A.1 Proof of Lemma 1 (Aggregation)

In describing the derivation, we omit the time subscript whenever it is not necessary. The firms within each product group choose the optimal prices, internalizing impacts on the group-level price indices. This leads to the equilibrium pricing $p_{jk} = \frac{\epsilon(s_{jk})}{\epsilon(s_{jk})-1} e^{a^L_{jk}/1-\sigma}$, where $\epsilon(s) = \left(\frac{1}{\theta}s + \frac{1}{\sigma}(1-s)\right)^{-1}$. Plugging this pricing formula to the demand curve, we have the relative market share as in the main text.

Using the equilibrium markups, we write the price index at the group level given brand gap b as $P(b) = \left(\frac{e^{a+b}\mu_1(b)^{1-\sigma}+\mu_2(b)^{1-\sigma}}{1+e^{a+b}}\right)^{1-\sigma}$. From this definition of price index, we aggregate the demand at the group level as $C(b) = \frac{P(b)^{-\theta}}{\mathbf{P}^{1-\theta}}\mathbf{C}$. From the definition of the labor productivity and the markup, we can write $P(b) = \frac{P(b)}{M(b)}$. Thus the demand of the group can be written as $C(b) = \frac{(Z(b)/M(b))^{-\theta}}{\mathbf{P}^{-\theta}}\mathbf{C}$.

We now aggregate groups into the entire economy. First, the optimal labor supply condition from the household implies that the aggregate expenditure equals to the inverse of the disutility of work, normalized to 1, $\mathbf{PC} = 1$. Using our definition of group-level productivities and markups, we write the aggregate price index as $\mathbf{P} = \left(\int_b (Z(b)/M(b))^{1-\theta}\right)^{\frac{1}{1-\theta}}$. From the labor supply condition $\mathbf{C} = \frac{1}{\mathbf{P}}$ and from the definition of $\mathbf{Z}$ and $\mathbf{M}$, we can write $\mathbf{C} = \frac{\mathbf{Z}}{\mathbf{M}}$. Lastly, using the definition of $\mathbf{M}$, we can write $\mathbf{L}^P = \frac{1}{\mathbf{M}}$.

A.2 Proof of Lemma 2

Because the step size of reallocation is fixed at Δ and the reallocation intensity is fixed at $\bar{\lambda}$, the only endogenous decision is the direction of reallocation. In this simple case, the joint surplus between the frontier and the vintage firms solves the following:

$$(\rho+\gamma+\lambda)\Omega(b) = \omega(b) + \gamma v(b_0) + \lambda \max\{\Omega(b+\bar{\Delta}), \Omega(b-\bar{\Delta})\}.$$

We re-write the equation as:

$$\Omega(b) = \frac{\omega(b)+\gamma v(b_0)}{\rho+\gamma+\lambda} + \frac{\lambda}{\rho+\gamma+\lambda}\max\{\Omega(b+\bar{\Delta}), \Omega(b-\bar{\Delta})\}.$$

This mapping satisfies the Blackwell sufficient conditions and is a contraction mapping. From the contraction mapping theorem, it suffices to guess and verify the unique solution. We start with the guess that the decision rule as in the lemma.

Because $\omega(b)$ is symmetric around $-a$, we conclude that $\Omega(b)$ is also symmetric around $-a$ under the guessed decision rule. With this guess, for $b > -a$, the joint surplus solves

$$\Omega(b) = \frac{\omega(b) + \gamma v(b_0)}{\rho + \gamma + \lambda} + \frac{\lambda}{\rho + \gamma + \lambda}\Omega(b + \bar{\Delta}).$$

Differentiating the equation we get:

$$\Omega'(b) = \frac{\omega'(b)}{\rho + \gamma + \lambda} + \frac{\lambda}{\rho + \gamma + \lambda}\Omega'(b + \bar{\Delta}).$$

As $b \to \infty$, this equation implies $\lim_{b\to\infty} \Omega'(b) \propto \lim_{b\to\infty} \omega'(b) > 0$. By recursively applying this equation, we conclude $\Omega'(b) > 0$ for $b > -a$. Using the same argument, we conclude $\Omega'(b) < 0$ for $b < -a$. Together with the symmetry, monotonicity implies that the guessed decision rule is optimal.

A.3 Proof of Lemma 3

Because the step size is fixed, the brand gap of any product group can be written as $b = b_0 + i\bar{\Delta}$, where i is the number of reallocation events that happened. Denote g_i the probability mass of groups that experienced i reallocation events, we write out the Fokker–Planck equation as:

$$0 = -\gamma g_i + \lambda g_{i-1},$$

which has the solution $g_i = \left(\frac{\lambda}{\lambda+\gamma}\right)^i$.

A.4 Planner's Solution

The planner maximizes the representative household's utility, given the resource constraints. The optimization problem of the planner is as following:

$$\mathbf{W}^* = \max_{\lambda(b),\iota(b)\in\{-1,1\}} \int_0^\infty \int_{-\infty}^\infty e^{-\rho t}\left[\log \mathbf{Z}_t \mathbf{L}^P - \mathbf{L}^P - \mathbf{R}_t\right] dt, \tag{A1}$$

s.t.

$$\dot{g}(b) = -\lambda(b)g(b) - \frac{\chi^2}{2}g''(b) + \lambda^+(b-\Delta)g(b-\Delta) + \lambda^-(b+\Delta)g(b+\Delta). \tag{A2}$$

The derivation of necessary conditions follows the method in Nuño and Moll (2018), where we set up the Lagrangian incorporating the constraint and derive the necessary condition. The HJB of the planner in the main text is the necessary condition that will set the variation to zero.

We now detail the derivation of the firms' decisions under a size-dependent policy. Consider an alternative planner that only has access to the market share data, and interpret it as productivity. The planner understands that there is variable markup, so it can reverse the demand system to recover the compounded productivity

$$z_1 = \left(\frac{e^{a+b}}{1+e^b}\right)^{\frac{1}{\sigma-1}}, \; z_2 = \left(\frac{1}{1+e^b}\right)^{\frac{1}{\sigma-1}}$$

The planner thus views the data through a demand system

$$\frac{\sigma-1}{\sigma}\log C = \log\left[c_1^{\frac{\sigma-1}{\sigma}} + c_1^{\frac{\sigma-1}{\sigma}}\right]$$

while the first firm has productivity z_1 and the second firm has productivity z_2. To offset the markup, the planner set:

$$T'(q) = -p'(q)\,q$$

we leave the constant of this transfer function as T_0 and integrate:

$$T(q) = T_0 + \int_0^q p'(x)x dx = T_0 - p(q)q + \int_0^q p(x)dx$$

where the second equality comes from integrating by part. Under the efficient allocation, the price equals the marginal cost. Thus

$$p(x) = \frac{x^{-\frac{1}{\sigma}}}{x^{\frac{\sigma-1}{\sigma}} + z_{-}^{\sigma-1}}$$

Integrating and imposing price equals marginal costs

$$\begin{aligned}
T(q) = T_0 - p(q)q + \int_0^q p(x)dx &= \frac{\sigma}{\sigma-1}\log\frac{q^{\frac{\sigma-1}{\sigma}} + z_{-}^{\frac{1}{\sigma}} + n}{z_{-}^{\frac{1}{\sigma}} + n} \\
&= T_0 - \frac{q^{\frac{\sigma-1}{\sigma}}}{q^{\frac{\sigma-1}{\sigma}} + z_{-}^{\sigma-1}} + \int_0^q \frac{x^{-\frac{1}{\sigma}}}{x^{\frac{\sigma-1}{\sigma}} + z_{-}^{\sigma-1}} dx \\
&= \frac{q^{\frac{\sigma-1}{\sigma}}}{q^{\frac{\sigma-1}{\sigma}} + z_{-}^{\sigma-1}} + \frac{\sigma}{\sigma-1}\log\frac{q^{\frac{\sigma-1}{\sigma}} + z_{-}^{\frac{1}{\sigma}}}{z_{-}^{\frac{1}{\sigma}}}
\end{aligned}$$

In the pricing equilibrium with such a transfer function:

$$\pi_j^s = \frac{\sigma}{\sigma-1}\log\frac{\frac{1+e^{a+b}}{1+e^b}}{z_j^{\sigma-1}}$$

Summing the payoff of the two firms under the subsidy we get the joint profit function as in the main text.

B Data Appendix

This section addresses the set of data sources relevant for the analysis and the data examples that motivate our investigation. Appendix B.1 expands on the details of the merge across datasets.

B.1 Data and Definitions

Merge. Our main merge links USPTO Trademark data with RMS Nielsen Scanner data. We proceed by linking firms and products separately. Our merge matches over 80% of sales-weighted products. Some problems still emerge with short-names. We use "tokens" and fuzzy matches to deal with the names.

Firms and products follow similar procedures and we discuss them in turn.

Datasets. We use four "parent" datasets in our study. We make use of brand-level data, which has brand ID, firm ID, product group code, sales, prices, and year. We make use of firm-level data which contains firm-level sales by group and year. We also make use of customer-level data, which has household ID, product-level detail, and year. We finally also make use of retail-level data for our local market regressions.

Product Definition. A product is defined as a UPC code (12-digit identifier) linked to the *Nielsen* parent firm. Products lie underneath the umbella of a brand. Brands also have brand codes which correspond to an umbrella aggregate across all brands in the main maturity specification to avoid brand $\times$ product variation. We study the life cycle of both products and brands in Appendix C.2.

Transaction Definition. Transactions are defined at both the Nielsen and USPTO level. The reason we define transactions using both is as follows. We note that when we plot the results applying only USPTO transaction information we find as follows. Multiple serial numbers per brand.

Firms. For matching firms, we first standardize on a large set of firm tags, eliminating common firm words, e.g. "CORP", "INC", "ESTABLISHMENT").[1] We then take the cleaned and standardized name and match according to a tokenized bigram matching procedure.

Brands. By focusing on brands, we direct our attention to long-running products held by firms. USPTO Trademark data provides the "tm_name" or the name associated with a registered trademark. RMS Nielsen follows a similar format, which has a "brand_name". We join the two by employing a token name matching. For brand names, there are no further removals of tokens beyond the firm-level analysis.[2] For brand age, we focus on the "prior" brand, as in the broader brand umbrella of the production. For transacted brands, we observe the level of the transaction and focus on this.

Transactions. For this paper, we use transaction data to map to brand reallocation. This occurs when a parent company of a given brand is different in period t from $t-1$. We use transactions in both USPTO and RMS Nielsen Scanner data. Overall, we get approximately 20% of brand transactions from USPTO Trademark data and 80% of transactions from RMS Nielsen. While there are more transactions observed in trademark data, there are some within firm transactions we drop, as we generate a text similarity threshold above which we do not consider transactions. Further, some transactions in USPTO do not have companies that are both identified in the RMS Nielsen Scanner Data. We also append transactions manually that are listed in Refinitiv M&A data and matched to firms listed. Approximately 2% of our transactions are manually imputed.

[1] The full list is here ('AB', 'AG', 'BV', 'CENTER', 'CO', 'COMPANY', 'COMPANIES', 'CORP', 'CORPORATION', 'DIV', 'GMBH', 'GROUP', 'INC', 'INCORPORATED', 'KG', 'LC', 'LIMITED', 'LIMITEDPARTNERSHIP', 'LLC', 'LP', 'LTD', 'NV', 'PLC', 'SA', 'SARL', 'SNC', 'SPA', 'SRL', 'TRUST', 'USA', 'KABUSHIKI', 'KAISHA', 'AKTIENGESELLSCHAFT', 'AKTIEBOLAG', 'SE', 'CORPORATIN', 'GROUP', 'GRP', 'HLDGS', 'HOLDINGS', 'COMM', 'INDS', 'HLDG', 'TECH', 'GAISHA', 'AMERICA', 'AMERICAN', 'NORTH', 'OPERATIONS', 'OPERATION', 'DIVISION', 'COMPAGNIE','INTERNATIONAL', 'NORTH AMERICA', 'InBev').

[2] Standardizations include removing any relevant firm names as discussed in the firms section, but does not do any further standardizations and tracks the token grams within each brand name.

C Empirical Appendix

This section explores some additional evidence on a couple core empirical messages on brands and branding, addresses references in the main text, and discusses the robustness of our empirical analysis. We apply broader data from the USPTO to indicate the fact that large firms build large portfolios of brands and their acquired brands drive a larger share of their portfolio. We then discuss the product life cycle with reference to the literature and discuss the integration of the product life cycle with our firm-level analysis. In each case, we explore the robustness of our results to varying definitions.

We focus on the role of reallocation in the firm size distribution in Appendix C.1, returning to the study of the sources of concentration, and evaluate the robustness of the empirical results on firms. We also discuss the types of reassignment in the trademark data, which is in part a please for further investigation on the sources and implications of trademark reallocation. We expand on the brand maturity and life cycle in Appendix C.2, focusing on the interaction of age and sales, and the evidence for the importance of product maturity and sales dispersion over time. We further discuss our connection to the literature on the product life cycle and then turn to the robustness of product-level results. We then discuss the identification of brand and firm components of firm-level sales variation in Appendix C.3. Finally, we expand on the event studies with both different measures and various other outputs (such as customers and retail expansion) in Appendix C.4.

C.1 The Broad Prominence of Reallocation

This section develops extensions to Section 3 that focused on the importance of reallocation. The main message of this section is that trademark and brand reallocation plays an important role across an array of industries, it also is an important driver of firm sales variation, and leads to significant market share for leading firms.

USPTO Trademark Reassignment. The most reliable long-term data source for brand reallocation is USPTO Trademark data. Our focus in this paper is particularly on reallocation due to either pure reassignment (e.g. ownership transfer) or mergers & acquisitions. In this section, we discuss the general contours of the trademark data when it comes to reallocation of ownership. There is significant reallocation in the data, but some reallocation does not fall under the specific "merger" or "reassignment", but instead is linked to name changing, collateral, and other corrections and adjustments.

Table C1 splits the different transactions in the data into their different groupings. Most transactions in the data are available from 1970-2018. We order the transaction type by largest share of transactions. However, each transaction may contain a bundle of trademarks (e.g., transfer of ownership of "Odwalla" may be bundled with various sub-brands of the core brand Odwalla). For example, in the case of "Security Interest" (or collateral), note that on average a larger number of brands are involved in the pledged bundle.

While our main focus in this paper has been mergers and reassignments, we note the richness of the data on multiple margins. Name changes are frequent, as firms may attempt to retool but maintain brand loyalty. Further, as noted previously, trademarks are often used as collateral. While Security Interest transactions are a small share of overall exchanges (around 10%), they make up almost 25% of all trademarks in exchanges. However, without transfer the firm may continue to operate these product lines. The benefit of focusing on mergers and reassignments is the reallocation of ownership and management

Table C1: Summary Statistics on Trademarks from USPTO

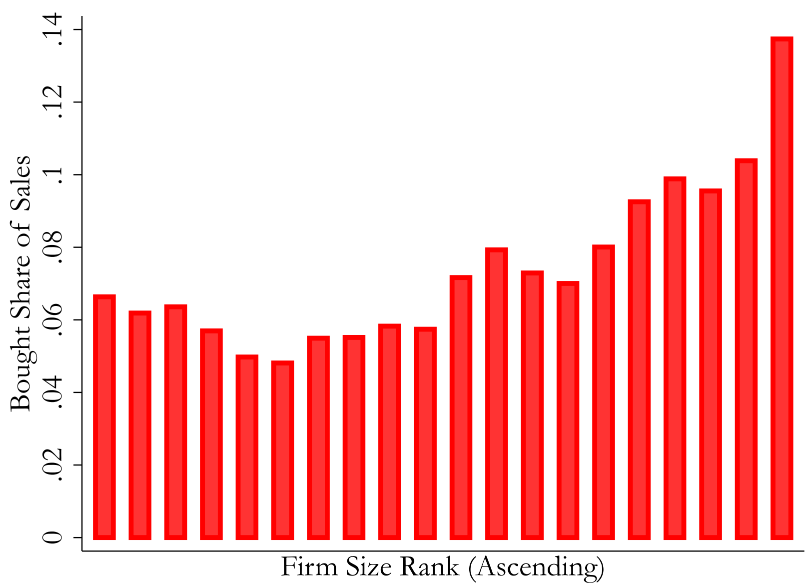
	Transaction Count	Trademark (TM) Count	TM/Transaction	Transaction Share	TM Share
Reassignment	478442	1.54M	3.21	**0.523**	0.345
Name Change	200767	795465	3.96	**0.219**	0.178
Security Interest	101280	1.10M	10.91	**0.111**	0.248
Merger	46610	287001	6.16	**0.051**	0.064
Correction	23500	119017	5.06	**0.026**	0.027
Other	64456	615334	9.55	**0.070**	0.138
Total	915055	4457996	4.87	1	1

Note: This table describes the category of each transaction in USPTO and orders them by their share of total transactions. Source: USPTO.

across firms, but we hope to see further research on these margins.

Firm Size and Brand Reallocation. Consistent with our main message, larger firms have more holdings from reassignment. Figure C1 shows this pattern with respect to sales in Nielsen Scanner Data. We plot the share of sales from bought brands against the ascending rank bins (running from 1-20) of the firm size in sales.

Figure C1: Contribution of Buying to Sales Share

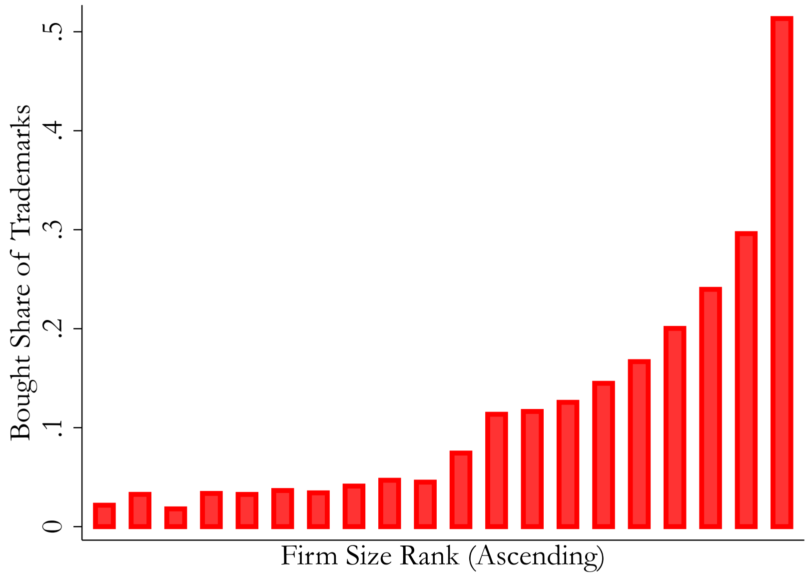


Notes: Share of total sales from poached brands. Source: RMS Nielsen and USPTO.

We find that the highest-selling firms have almost 3-times as much acquired share of sales that a median firm, indicating that the pattern we find in the Trademark data on its own is consistent in the sales-share data. We observe this in both RMS Nielsen Scanner data and in USPTO Trademark data. Turning to USPTO data, we find the results are even more stark. Large firms tend to carry bought trademarks as a much larger share of their portfolio. This is noted in Kost et al. (2019), and can be seen in Figure C2.

As noted in the main text, we perform a variance decomposition of the sources of changes in market

Figure C2: Contribution of Buying to Trademark Stock



Notes: Share of trademarks that have been bought from other firms as a share of total trademark holdings. Source: USPTO.

share at the firm level. In the main text, we weighted by product group size, but not firm size. We find that when weighting by firm size the quantitative magnitudes of the shares are similar, though the total variance is lower. Table C2 reports the variance decomposition, mirroring Table 3 in the text.

Table C2: Variance Decomposition of Firm Market Share Growth, Weighted by Firm Size

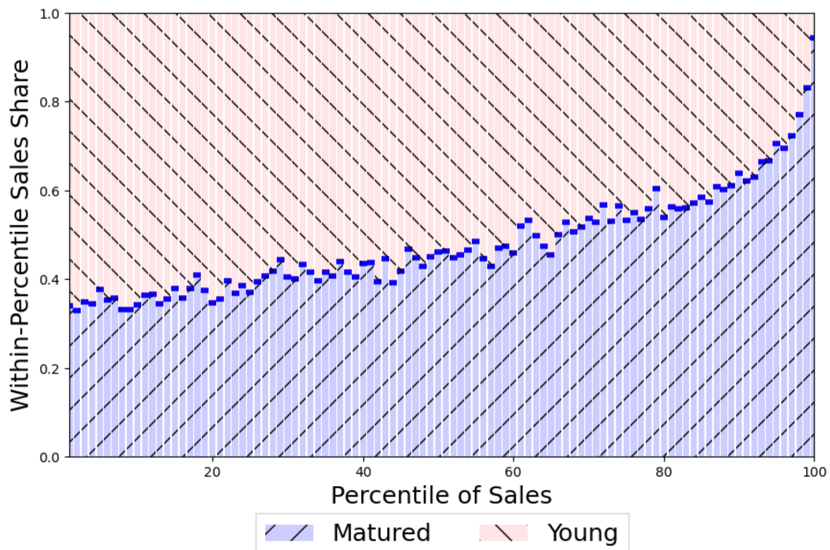
	Total Variance	New Brands	Incumbent Growth	Reallocation
All Firms	0.12	4%	69%	28%
Firms >0.01% Share	0.11	4%	69%	29%
Firms >0.1% Share	0.10	4%	67%	30%
Firms >1% Share	0.08	4%	62%	32%
Firms >5% Share	0.06	5%	59%	33%

Table C2 is consistent with Table 3 and the main message of this section. The table shows the broad prominence of reallocation and incumbent growth for firm market share, with larger firms relying more on reallocation for their firm size dispersion.

C.2 Brand Maturity

This paper treats brand capital as a transferable form of intangible that would otherwise take time to produce. When firms want to acquire customer capital, they can build it directly, which is costly in time, or acquire it. The core tenets of this point can be found in recent macroeconomic literature. Gourio and Rudanko (2014) and Foster et al. (2016), among many others, have noted that customer capital is not built in a day. This complements the importance of reallocation of existing brands for economic analysis. This section expands on this point by describing the lifecycle of brands. In particular, we note the time-to-build nature of brand capital. By looking at trademark data and Nielsen data, one can observe the importance of senior brands. Figure C3 takes data from 2016. We plot the brand percentile in terms of overall sales on the x-axis. On the y-axis, we plot the share of sales in this group that belongs to brands older than 10 years and brands younger than 10 years.[3]

Figure C3: Brand Percentile and Maturity



Note: This figure shows the sales share within a percentile bin of products, split by those born before 2006 ("Matured") and after 2006 ("Young"). Source: RMS Nielsen Scanner Data.

For brands created in 2006 and earlier, they maintain large sales share into the future. By 2016, those brands are still dominant in the top 1% of brands. Within the top 1% of brands, brands created before 2006 make up 92% of sales. Overall, old brands make up over 70% of sales, but only about 1/3rd of products. For the median brand in terms of sales, older brands make up less than half (38%) of total sales.

The dominance of mature brands could come from two forces. First, if few brands achieve such large sales, there may be a selection process. Young brands have less of a chance than old brands to have high customer capital, as the brands that survive to maturity must have a high quality draw. The composition only selects for the best. Second, brands could increase their sales over the life cycle such that only mature brands have significant sales share. We aim to understand this by linking a brand to its specific age. By employing the USPTO-Nielsen merge, we are able to extend this life cycle beyond the current work in the literature, but we review our connection to the current literature benchmarks here.

Literature Benchmark: The Product Life Cycle. As discussed in the main text, our findings on the brand life cycle are significantly longer than the life cycle discussed in recent work (e.g. Argente et al., 2018). Here, we crosswalk our results to existing work on the product life cycle to benchmark where we diverge. Argente et al. (2018) focus on the life cycle of products applying Nielsen Scanner Data. This work is able to identify new products and brands and document their life cycle patterns. However, it is not able to link brands and products to their history, and is thus unable to speak to the longer time horizon of persistent brands. We perform similar life cycle regressions to this current paper in the literature, in particular focusing on defining age in two different ways, to ensure the differences in the age profile does not simply

[3]We omit brands with less than $1000 in sales over an entire year, to have only brands that at least have a product line.

come from applying a dataset with different age measures. Equation (A3) presents the regression:

$$\log y_{it} = \alpha + \sum_{a=0}^{4} \beta_a D_a + \gamma_b + \lambda_t + \epsilon_{it} \tag{A3}$$

Where the coefficients of interest are the coefficients on age (β_a) with controls for cohort and time effects (and an adjustment on cohort from Deaton, 1997). Table C3 engages in the same specification as Argente et al. (2018) in the UPC data (panels 1 and 2) and Trademark merged data (panels 3 and 4) respectively.

Table C3: Log Sales, by Nielsen and Trademark Age

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales
Age 1	0.939***	1.095***	0.917***	0.953***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 2	0.857***	1.159***	1.019***	1.060***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 3	0.632***	1.016***	0.834***	0.832***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 4	0.169***	0.644***	0.412*	0.488***
	(0.00)	(0.00)	(0.00)	(0.00)
N	668993	89203	3402	4136
R^2	0.138	0.179	0.256	0.050
Variation	UPC	Brand-Group	TM Brand	TM Brand-Group

p-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Notes: Balanced Panel Life Cycle Regressions of Log Sales on Age, utilizing different age sources and different variation. Source: USPTO Trademark and RMS Nielsen

We find that sales increase in early years (often increasing from Age 1 to Age 2), but decline steadily post Age 2. This is broadly consistent with Argente et al. (2018). Note that while at the level of brands and trademarks there are significantly fewer observations, the same general pattern holds. This indicates how age is picking up something similar in our context, yet due to the broader horizon of historical data we are able to connect brands to their histories, indicating a significantly longer brand life cycle than found in Argente et al. (2018). We also show here similar general trends as in the main text when we evaluate the life cycle of products, controlling for brand-firm-group level.

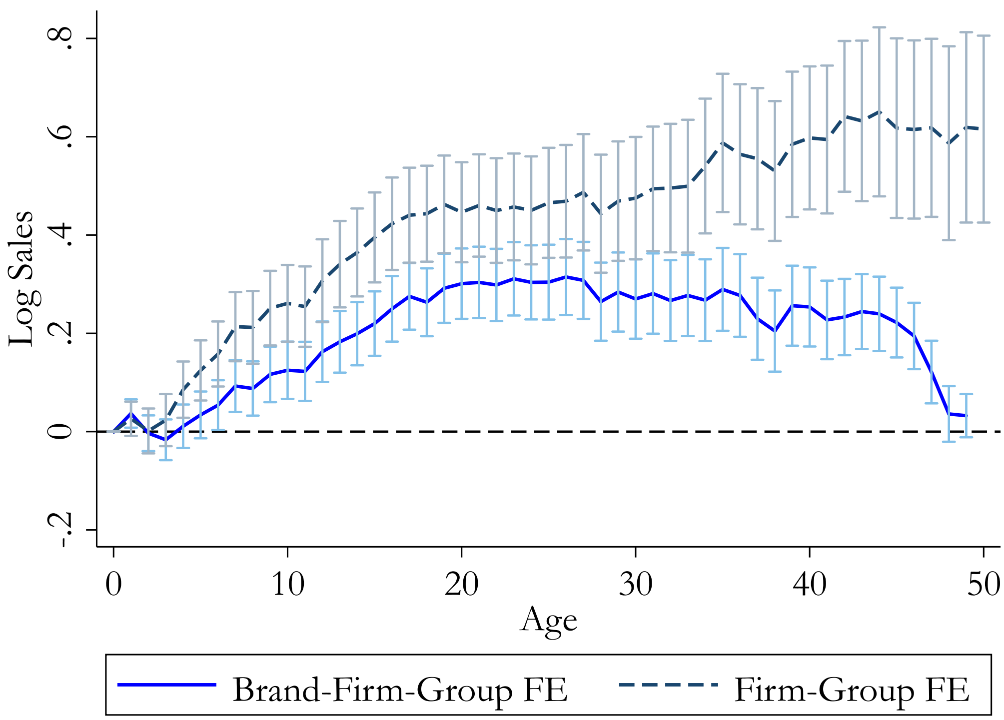
Figure C5 evaluates the life cycle profile within a given product group code. We follow the regression in the main text, except in prices we weight by sales share. Equation (A4) illustrates the structure of the regression.

$$\log y_{ijkt} = \alpha + \sum_{a=1}^{50} \beta_a D_a + \gamma_b + \lambda_t + \theta_{ikj(i)} + \epsilon_{ijkt} \tag{A4}$$

The regression in Equation (A4) considers the sales and prices of brand i with firm j in group k at time t, $\log y_{ijkt}$ as a function of a constant (α), brand age indicators from 1 to 50, D_a, and fixed effects for cohort (γ_b) and time (λ_t).[4] The $\theta_{ikj(i)}$ indicates a brand-group or firm-group fixed-effect. Figure C5 plots the regressions by age coefficient β_a.

[4]Given the linear relationship between age, time, and cohort, we follow a method developed by Deaton (1997) to correct for this issue. The normalization orthogonalizes the cohort trends such that growth components move with age and time effects.

Figure C4: Life Cycle Regressions



Note: Plots of log sales on age regression coefficients, controlling for brand-group and controlling only for firm-group. 95% confidence intervals plotted alongside coefficients. Source: RMS Nielsen and USPTO.

Figure C5 is consistent with the main facts from Section 3.2. We note that the inverted-U profile is still persistent within group, though with a slightly lower peak than in the brand's overall life cycle. We also note that the life cycle of prices shows on average somewhat minimal activity for the brand across age. This means that the strategic pricing firms engage in does not appear to be correlated with age, though as we have noted from events there are shifts in prices, consistent with previous evidence in the literature.

C.3 Brand and Firm Decomposition

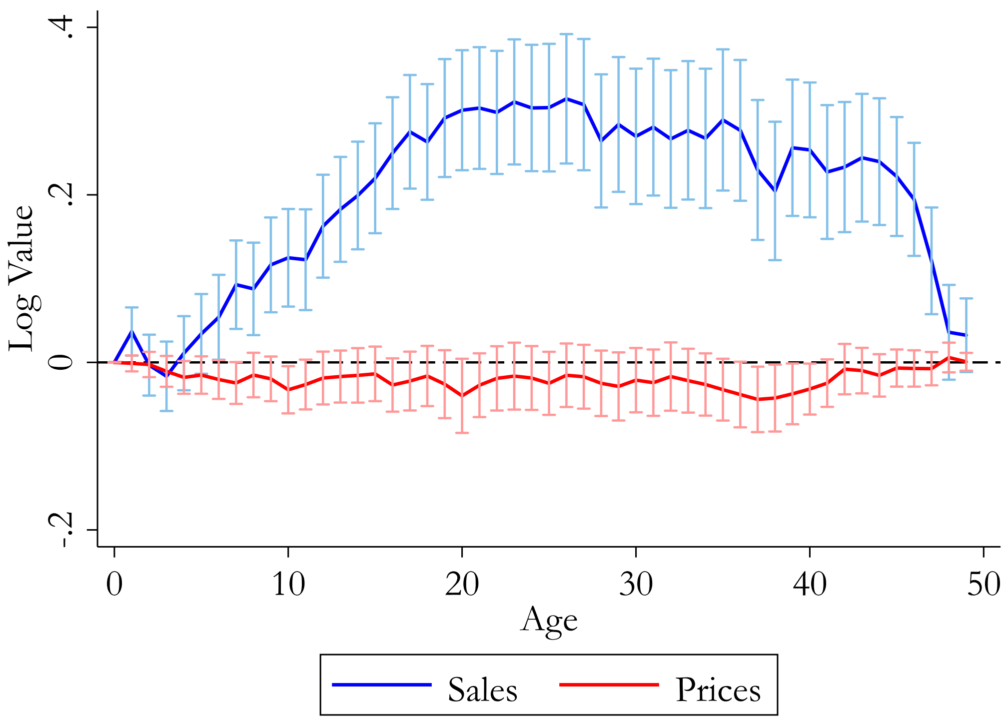
One of the main messages of this paper is the importance of the brand as an independent and transferable form of intangible capital. In our event studies, we find changes in sales associated with a brand transaction, but also significant persistence. In this section, we expand on the separable brand and firm component of overall sales by applying econometric methods for teasing out the contribution of each to the overall sales at the firm level.

To understand the contribution of the firm and the brand to sales, we build on work that attempts to separate the firm and worker component of wages as in Abowd et al. (1999) and Bonhomme et al. (2019).

Bonhomme et al. (2019) proceed in two steps, which we somewhat mirror. First, the authors split firms into different types using a *k*-means clustering algorithm. Second, the authors identify off of movers across firm types the firm and worker effect (in our case, the firm and brand effect). If one ignores the brand, all variation would be assigned to the sales share of the firm. In the transaction, we can identify how the brand itself has persistence even when reallocated across firms.

In our case... Our main fact on the brand-firm decomposition expands on our event studies by looking

Figure C5: Life Cycle Regressions, Prices and Sales within Group



Note: Plots of log sales and prices on age regression coefficients, controlling for brand-group, as in Equation (A4). 95% confidence intervals plotted alongside coefficients. Source: RMS Nielsen and USPTO.

at the overall sources of market share at the firm level.

Table C4: Abowd et al. (1999) variance of firm log sales explained by brand

	10 bins	50 bins	100 bins
Brand	62%	61%	61%
Firm	18%	19%	19%
Brand × Firm	20%	20%	20%
Share Reallocated by Period	1.4% (5.6% overall market share change)[5]		
Top Firm Share	33.4%		
Median Firm Share	0.005%		

Regardless of how we split firm size, we see significant sales variation coming from the brand-level drivers of firm size. This is consistent with brands being a central component of firm market share. Given the fact that brands are tradable, this indicates a huge component of firm value lies in a tradable asset. This bolsters our main point on the importance of brands, in particular speaking to Fact 1.

C.4 Event Study Details

In the main paper, we focused on the responsiveness of sales and prices to reallocation events. Here, we expand the discussion to explore further the implied and noted mechanisms in the main text.

Here, we focus on a couple of components of the event study. First, we expand on the definition of the event study. We then focus on the prices and sales of brands at "top" firms. Finally we unpack the mechanics of reallocation: the role of customer acquisition, the nature of retail expansion, and the effects at the UPC level.

Event Study Definition. Our event studies focus on transactions across firms in the data. For an observed transaction, both the buyer and the seller must exist in the data. We employ a balanced panel with seven periods. Given we use data from 2006–2018, we must restrict our event study analysis to brand transactions from 2009–2015. Due to some of the restrictions on our data, we focus on a broader definition of leading firms and flows from low-type to high-type firms. We explore the robustness of event studies depending on our characterization of an event study and definition of firm type.

Prices and Sales at Top Firms. In this section, we explore varying the definition of a top firm to understand the differences in predicted sales. Table C5 focuses on the robustness of the higher log sales at larger firms. We see that larger firms tend to show higher sales of the same brand.

Table C5: Log Sales Conditional on Holding Firm, Trademark Age Fixed Effects

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales	(5) Log Sales	(6) Log Sales
Top 10 Overall	0.57***	0.55***				
	(0.000)	(0.000)				
Last Period Top 10			0.59***	0.69***		
			(0.000)	(0.000)		
Top 10 in 2006					0.47***	0.53***
					(0.000)	(0.000)
N	441300	3972	441300	3972	441300	3972
R^2	0.844	0.741	0.844	0.735	0.844	0.740
Weights	No	No	No	No	No	No
Restrictions	No	Only trans.	No	Only trans.	No	Only trans.

p-values in parentheses, clustered at brand-group level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Table C6 focuses on the robustness of the higher log prices at larger firms, focusing only on the merged sample. We note that the results directionally hold, but exhibit a higher variace.

Customer Access. What does brand reallocation do for firms? In line with the evidence from Section 3.2, brands provide customer access for a firm, and firms can acquire brands to acquire the customer base connected to the brand. It has been noted that consumers have persistent preferences over brands (Bronnenberg et al., 2012), but this has not been studied with brand reallocation. To understand whether an acquisition of brands acquires customers, we track what happens to the customers at a firm and a brand when a brand is reallocated across firms. We find that a reallocation event is associated with an addition of new customers to the firm. Table C7- C9 focus on this at the brand and UPC level, asking if a brand is reallocated, what happens to the customer that is currently consuming (exit) and not consuming (entry) the brand or product.

Table C6: Log Price Conditional on Holding Firm, TM age FE

	(1) Log Price	(2) Log Price	(3) Log Price	(4) Log Price	(5) Log Price	(6) Log Price	(7) Log Price	(8) Log Price
Top 10 Firm	0.33	0.26*			0.057	0.036		
	(0.177)	(0.062)			(0.170)	(0.403)		
Top 10 Firm in 2006			0.14	0.34*			-0.0091	0.029
			(0.350)	(0.088)			(0.869)	(0.516)
N	441300	3972	441300	3972	441300	3972	441300	3972
R^2	0.967	0.881	0.967	0.882	0.983	0.983	0.983	0.983
Weights	Total Wt.	Total Wt.	Total Wt.	Total Wt.	Period Wt.	Period Wt.	Period Wt.	Period Wt.
Restrictions	No	Only trans.	No	Only trans.	No	Only trans.	No	Only trans.

p-values in parentheses, clustered at brand-group level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Table C7: Customer-Level Entry and Exit at Brand Level

	(1) Entry	(2) Entry	(3) Entry	(4) Entry	(5) Exit	(6) Exit	(7) Exit	(8) Exit
Reallocation Event	0.023*	0.023*	0.0076***		0.029**	0.027**	-0.0089***	
	(1.75)	(1.78)	(4.48)		(2.45)	(2.31)	(-4.82)	
Lag Reallocation Event				0.0085***				-0.0047***
				(3.37)				(-4.01)
N	12050391	12050015	12049974	12049974	11181462	11180993	11180959	11180959
Fixed Effect	Year	Year+HH	Year+HH+UPC	Year+HH+UPC	Year	Year+HH	Year+HH+UPC	Year+HH+UPC

t statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table C8: Customer-Level Entry at UPC Level

	(1) Customer Entry	(2) Customer Entry	(3) Customer Entry	(4) Customer Entry
Reallocation Event	0.025***	0.026***	0.016***	
	(4.47)	(4.64)	(5.25)	
L. Reallocation Event				0.015***
				(4.62)
Fixed Effect	Year	Year+HH	Year+HH+UPC	Year+HH+UPC
N	122868370	122868364	122704601	122704601

t statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Productive and Strategic Effects: Sales and Prices. We observe brand transactions in the data and ask how prices and sales respond.[6] To ensure a relevant comparison group, we link transacted brands to never transacted brands with similar age, sales trends, and product group codes to the focal brands in this setting.[7] Both transacted brands and placebo brands are active for 7 years (3 years before event, event period, 3 years after), ensuring a balanced panel.

After the event, both prices and sales move strongly, with sales moving more. With the increase in

[6] We follow the same measurement of log sales and log prices in both the observed regressions and the event studies.

[7] We engage in a CEM distance matching but also do Mahabonolous matching which coarsens the sales into 30 bins in the pre-period. Similar results are found in both cases.

Table C9: Customer-Level Exit at UPC Level

	(1) Customer Exit	(2) Customer Exit	(3) Customer Exit	(4) Customer Exit
Reallocation Event	0.0067	0.0067	0.0043**	
	(1.07)	(1.08)	(2.50)	
L. Reallocation Event				-0.0015
				(-1.08)
Fixed Effect	Year	Year+HH	Year+HH+UPC	Year+HH+UPC
N	128882857	128882856	128713685	128713685

t statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

prices, the results in Figure 4 provide evidence that both mechanisms, strategic acquisition and productive acquisition, could be at play.

Heterogeneity. While the average sales and price effects indicate an interesting tension between productive and strategic interaction, it masks a significant heterogeneity across transactions. Part of the motivating model presented indicated that depending on features of the market structure, such as productivity and the market share gap, some transactions may be efficient while others may be strategic. This tension can be found in the data, as some transactions exhibit what look like purely a productivity gain (only sales go up, and prices stay flat or decline), while others exhibit a purely strategic effect (prices go up, with a negative effect on sales). For instance, 14% of transactions exhibit a price change above the average and a sales change below the average. 18% exhibit a sales change above the average and a price change below the average.

Retail Expansion. This section address the nature of retail expansion for a brand when it is acquired by another firm. We focus on the corresponding change in unique retail chains when a brand is acquired. We perform simple regressions here, which are consistent with the results on sales above.

$$y_{it} = \alpha_0 + \alpha_1 post\_event + \Lambda_i + \Gamma_t + \epsilon_{it} \tag{A5}$$

Table C10 reports the estimated coefficient, $\hat{\alpha_1}$, in Equation (A5). We perform the regression on a unbalanced panel and balanced panel to show the effect on a larger number of observations (unbalanced) and to control for before and after symmetry (balanced).

We find a rise in the unique stores after the event. Though we do not use a comparison group here, we do find results broadly consistent with the sales growth in the event study.

UPC Analysis. Most of our analysis is done at the brand level. When we follow specific products, we find similar overall effects of the event study. We perform the following specification:

We return to our events studies at the Un two different outcome variables in our event studies: revenues and prices. We compare the reallocated brands to a group of similar brands within the same product group around the time of the reallocation event. These similar brands will be comparable brands that did not experience a reallocation event. We then estimate the following regression in Equation (A6),

Table C10: Retail Chain Regression, Pre- and Post-Event

	(1) Log Unique Chains	(2) Log Unique Chains
Post Indicator	0.079***	0.15***
	(0.020)	(0.040)
N	25142	3563
R^2	0.814	0.860
Sample Selection	Unbalanced Panel	Balanced Panel
Fixed Effects	Brand-Group and Year	Brand-Group and Year

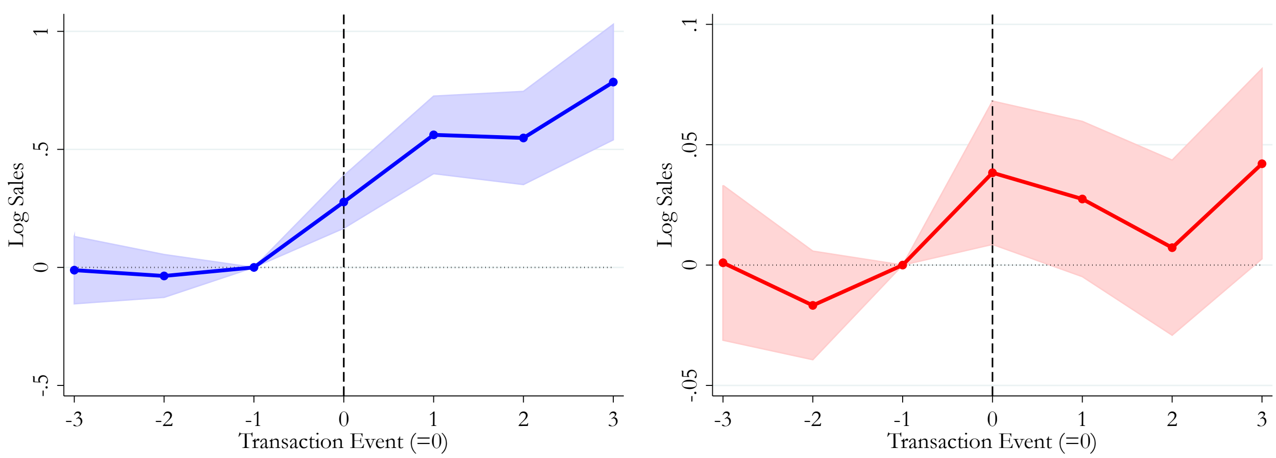
Standard errors in parentheses clustered at the brand-group level.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table documents two separate regressions on brands incorporating a balanced panel (3 years before event, 3 years after) and an unbalanced panel (all years before and after).

$$\log y_{ikt} = \sum_{\tau=-3}^{3} \zeta_\tau \times \mathbb{I}\{t=\tau\} + \sum_{\tau=-3}^{3} \alpha_\tau \times reallocated \times \mathbb{I}\{t=\tau\} + \xi_t + \theta_{ik} + \epsilon_{ikt}, \quad \text{(A6)}$$

Here, we plot the outcome focusing on matched UPCs rather than matched brands. This limits our analysis to brands which maintain the same UPCs before and after transaction. Figure C6 plots the results for sales (left panel) and prices (right panel). We find results consistent with Section 3. Sales and prices both increase.

Figure C6: Brand Transactions and Matched Pairs



(a) Sales × Transaction

(b) Prices × Transaction

Notes: Mahalanobis exact match coefficients. Match is made on pre-trend sales, exact product group, and exact year. 95% confidence interval standard errors clustered at the brand-group level. *Source: USPTO and RMS Nielsen.*